Spectrum Auction Design

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Market design

- Establishes rules of market interaction
- Economic engineering
  - Economics
  - Computer science
  - Operations research
- Applications
  - Matching
  - Auctions (matching with prices)

Market design fosters innovation

- Improving price information
- Enhancing competition
- Mitigating market failures

Applications

- Emission allowance auctions
- Airport slot auctions
- Spectrum auctions
- Electricity and gas markets
- Global financial crisis
- Green energy projects

Introduction

- Auction design
  - Government perspective (design)
  - Bidder perspective (strategy)
- Based on my experience
  - Researching auctions
  - Advising governments (12)
  - Advising bidders (31)

Application: Spectrum auctions

- Many items, heterogeneous but similar
- Competing technologies
- Complex structure of substitutes and complements
- Long-term market

- Government objective: Efficiency
  - Make best use of scarce spectrum
    - *Recognizing competition issues in downstream market*

Main points

- Enhance substitution
  - Product design
  - Auction design
- Encourage price discovery
  - Dynamic price process to focus valuation efforts
- Induce truthful bidding
  - Pricing rule
  - Activity rule

Simultaneous ascending auction

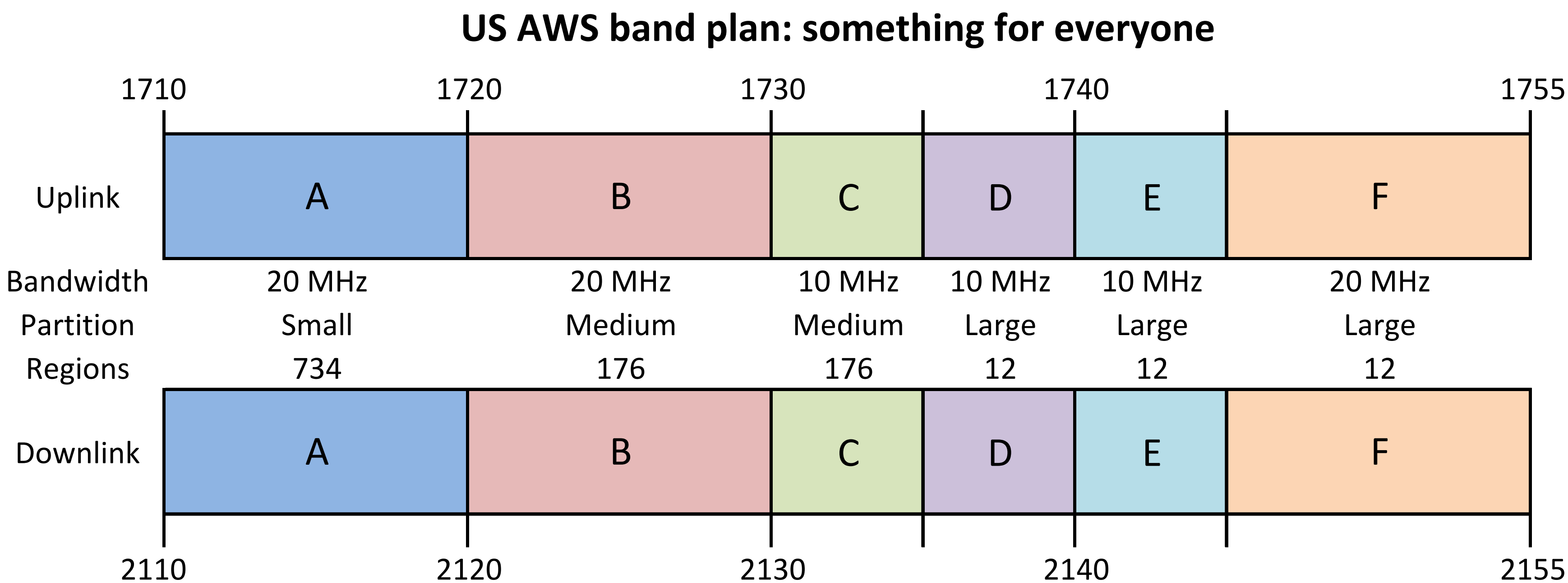
Simultaneous ascending auction

- Simultaneous
  - All lots at the same time
- Ascending
  - Can raise bid on any lot
- Stopping rule
  - All lots open until no bids on any lot
- Activity rule
  - Must be active to maintain eligibility

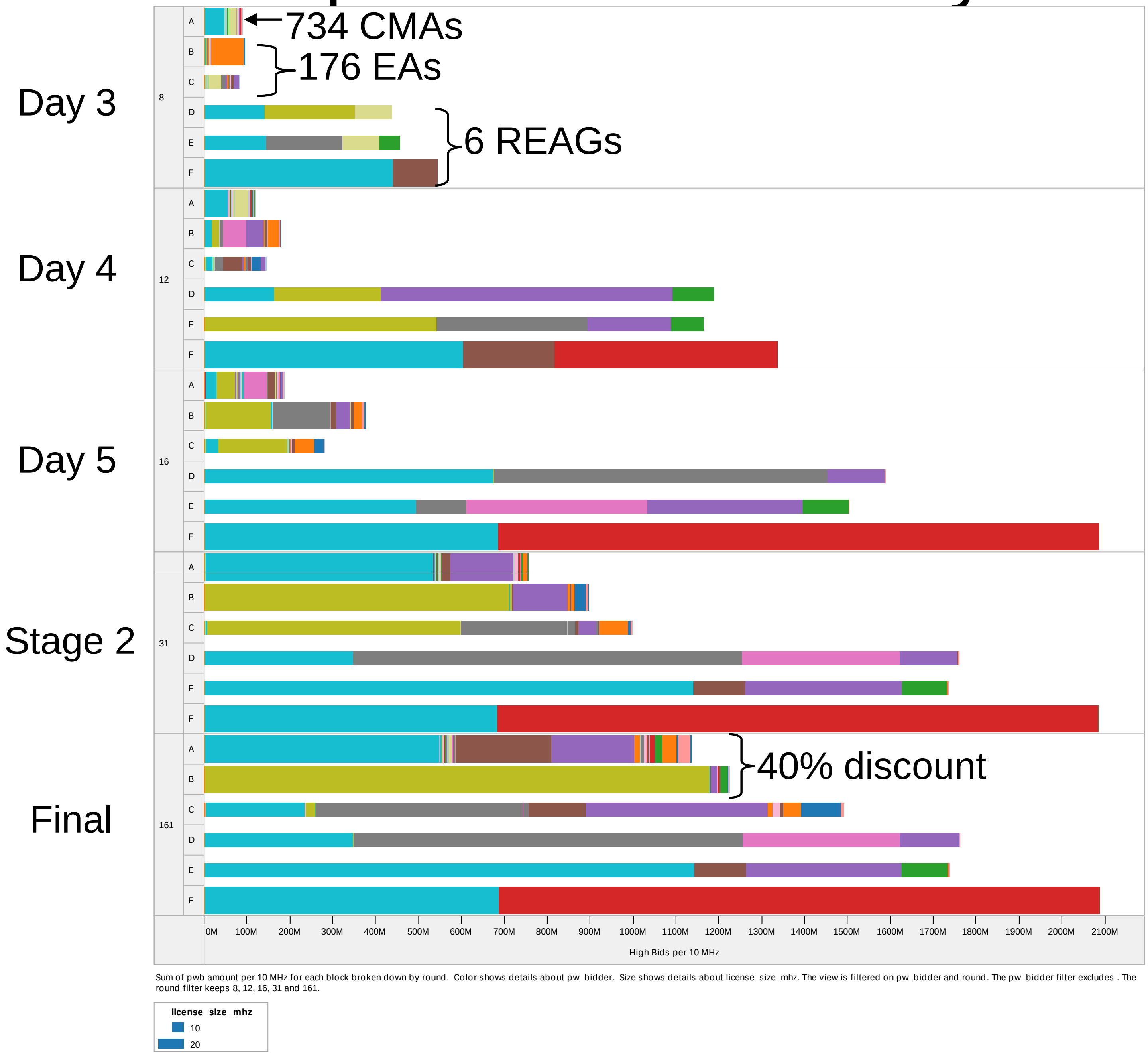
Simultaneous ascending auction

- Strengths
  - Simple price discovery process
  - Allows arbitrage across substitutes
  - Piece together desirable packages
  - Reduces winner’s curse
- Weaknesses
  - Demand reduction
  - Tacit collusion
  - Parking
  - Exposure
  - Hold up
  - Limited substitution
  - Complex bidding strategies

Limited substitution: US AWS
90 MHz, 161 rounds, $14 billion



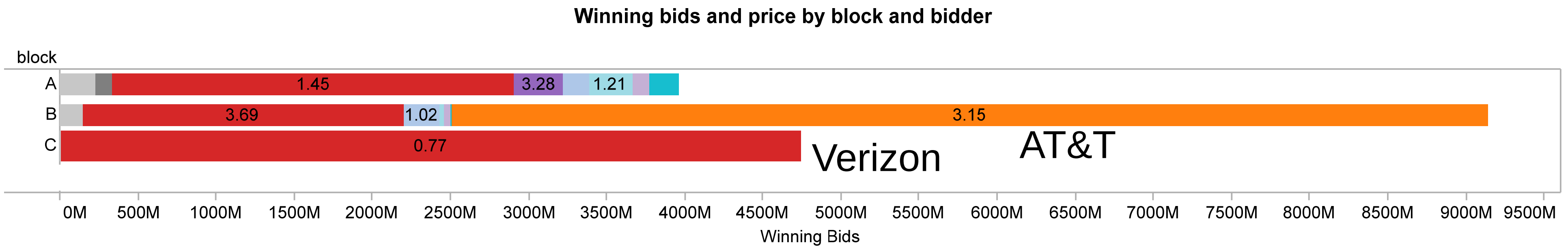
AWS price for 10 MHz by block



Sum of pwb amount per 10 MHz for each block broken down by round. Color shows details about pw_bidder. Size shows details about license_size_mhz. The view is filtered on pw_bidder and round. The pw_bidder filter excludes . The round filter keeps 8, 12, 16, 31 and 161.

Limited substitution: 700 MHz
62 MHz, 261 rounds, $19.6 billion

Block	A	B	C
Bandwidth	12 MHz	12 MHz	22 MHz
Type	paired	paired	paired
Partition	176	734	12
Price	$1.16	$2.68	$0.76



Verizon and AT&T won 85% of spectrum

A better way

Needed enhancements

- Anonymous bidding
- Generic lots
- Package bidding with clock
  - Porter-Rassenti-Roopnarine-Smith (2003)
  - Ausubel-Cramton (2004)
  - Ausubel-Cramton-Milgrom (2006)
- “Second” pricing
- Revealed preference activity rule

Package clock auction

- Auctioneer names prices;
  bidder names package
  - Price adjusted according to excess demand
  - Process repeated until no excess demand
- Supplementary bids
  - Improve clock bids
  - Bid on other relevant packages
- Optimization to determine assignment/prices
- No exposure problem (package auction)
- Second pricing to encourage truthful bidding
- Activity rule to promote price discovery

Example: AWS done right

Task	
Rule making	• 90 MHz paired spectrum; nine 2×5-MHz lots • Geographic partition: 176 Economic Areas
Preference elicitation	• Clock stage – FCC announces 176 prices – Each bidder selects best package – Prices rise where excess demand – Continues until no excess demand • Supplementary bids
Optimization	• Generic assignment; options for specific assignments (contiguous, min border issues)
Preference elicitation	• Top-up bids
Optimization	• Specific assignment

US AWS-3

- Two band plans proposed for 2020-2025 MHz and 2155-2180 MHz
  - TDD (unpaired) Five 5-MHz nationwide lots
  - FDD (asymmetric paired) One 5-MHz paired with five 5-MHz lots
- Should FCC offer paired or unpaired spectrum? LTE or WiMAX?
- Better solution: *Let auction decide!!*

UK
spectrum auctions

UK auctions

- ✓ 10-40 GHz: fixed wireless or backhaul
- ✓ L-Band: mobile broadcast
- 2.6 GHz: 4G mobile wireless (summer’09)
- Digital Dividend: 4G, mobile TV, DTT (’10)

<u>Requirements</u>

- Technology neutral
- Flexible spectrum usage rights
- Efficient assignment

UK 2.6 GHz auction proposal

- 190 MHz (38 lots of 5 MHz)
- How much paired vs. unpaired?

CEPT band plan from Electronic Communications Committee Decision (05)05

Type	Paired (FDD uplink)													
Lot	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Frequency	2500	2505	2510	2515	2520	2525	2530	2535	2540	2545	2550	2555	2560	2565

Type	Unpaired (TDD)									
Lot	1	2	3	4	5	6	7	8	9	10
Frequency	2570	2575	2580	2585	2590	2595	2600	2605	2610	2615

Type	Paired (FDD downlink)													
Lot	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Frequency	2620	2625	2630	2635	2640	2645	2650	2655	2660	2665	2670	2675	2680	2685

Let auction determine band plan

Increase in unpaired spectrum maintaining 120 MHz duplex spacing

Type	Paired (FDD uplink)									
Lot	1	2	3	4	5	6	7	8	9	10
Frequency	2500	2505	2510	2515	2520	2525	2530	2535	2540	2545

Type	Unpaired (TDD)													
Lot	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Frequency	2550	2555	2560	2565	2570	2575	2580	2585	2590	2595	2600	2605	2610	2615

Type	Paired (FDD downlink)										Unpaired			
Lot	1	2	3	4	5	6	7	8	9	10	15	16	17	18
Frequency	2620	2625	2630	2635	2640	2645	2650	2655	2660	2665	2670	2675	2680	2685

All unpaired spectrum

Type	Unpaired (TDD)									
Lot	1	2	3	4	5	6	7	8	9	10
Frequency	2500	2505	2510	2515	2520	2525	2530	2535	2540	2545

Type	Unpaired (TDD)									
Lot	11	12	13	14	15	16	17	18	19	20
Frequency	2550	2555	2560	2565	2570	2575	2580	2585	2590	2595

Type	Unpaired (TDD)									
Lot	21	22	23	24	25	26	27	28	29	30
Frequency	2600	2605	2610	2615	2620	2625	2630	2635	2640	2645

Type	Unpaired (TDD)							
Lot	31	32	33	34	35	36	37	38
Frequency	2650	2655	2660	2665	2670	2675	2680	2685

Key design choices

- Generic 5 MHz lots
  - Lots are perfect substitutes
- Package bids
  - No exposure problem
- Clock stage
  - How many paired? How many unpaired? Supply = 38
  - Continue until no excess demand
- Supplementary bids
  - Improve clock bids; add other packages
- Principal stage
  - Find value maximizing *generic* assignment and base prices
- Assignment stage
  - Contiguous spectrum
  - Top-up bid to determine *specific* assignment

Pricing rule

Pricing rule

- In clock stage? In assignment stage?
- Pay-as-bid pricing
  - Incentives for demand reduction, bid shading
- Bidder-optimal core pricing
  - Maximize incentives for truthful bidding

Bidder-optimal core pricing

- Minimize payments subject to core constraints
- Core = assignment and payments such that
  - Efficient: Value maximizing assignment
  - Unblocked: No subset of bidders prefers to offer seller a better deal

Optimization

- Core point that minimizes payments readily calculated
  - Solve Winner Determination Problem
  - Find Vickrey prices
  - Constraint generation method
    (Day and Raghavan 2007)
    - Find most violated core constraint and add it
    - Continue until no violation
- Tie-breaking rule for prices is important
  - Minimize distance from Vickrey prices

5 bidder example with bids on {A,B}

- $b_1\{A\} = 28$
- $b_2\{B\} = 20$
- $b_3\{AB\} = 32$
- $b_4\{A\} = 14$
- $b_5\{B\} = 12$

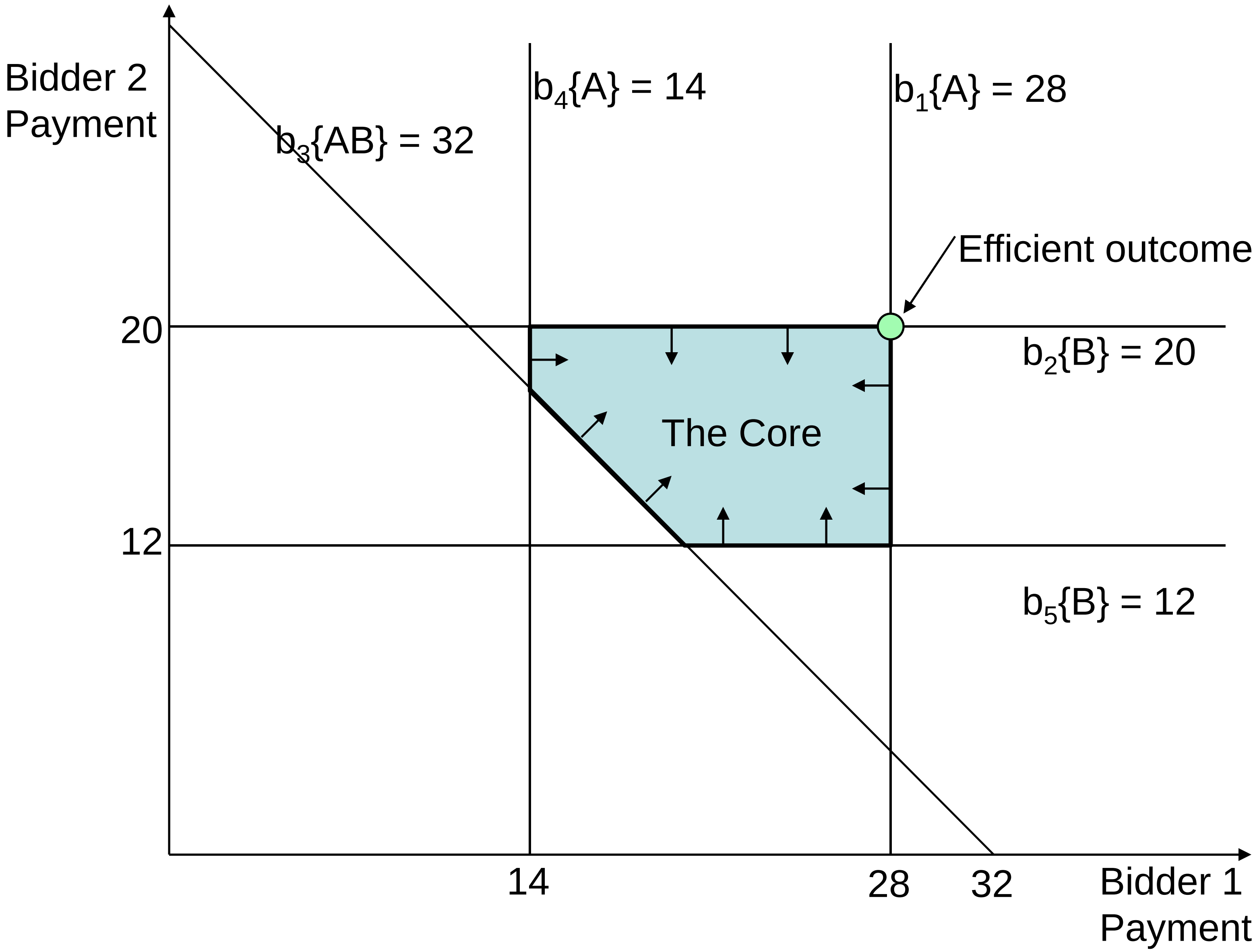
Winners

Vickrey prices:

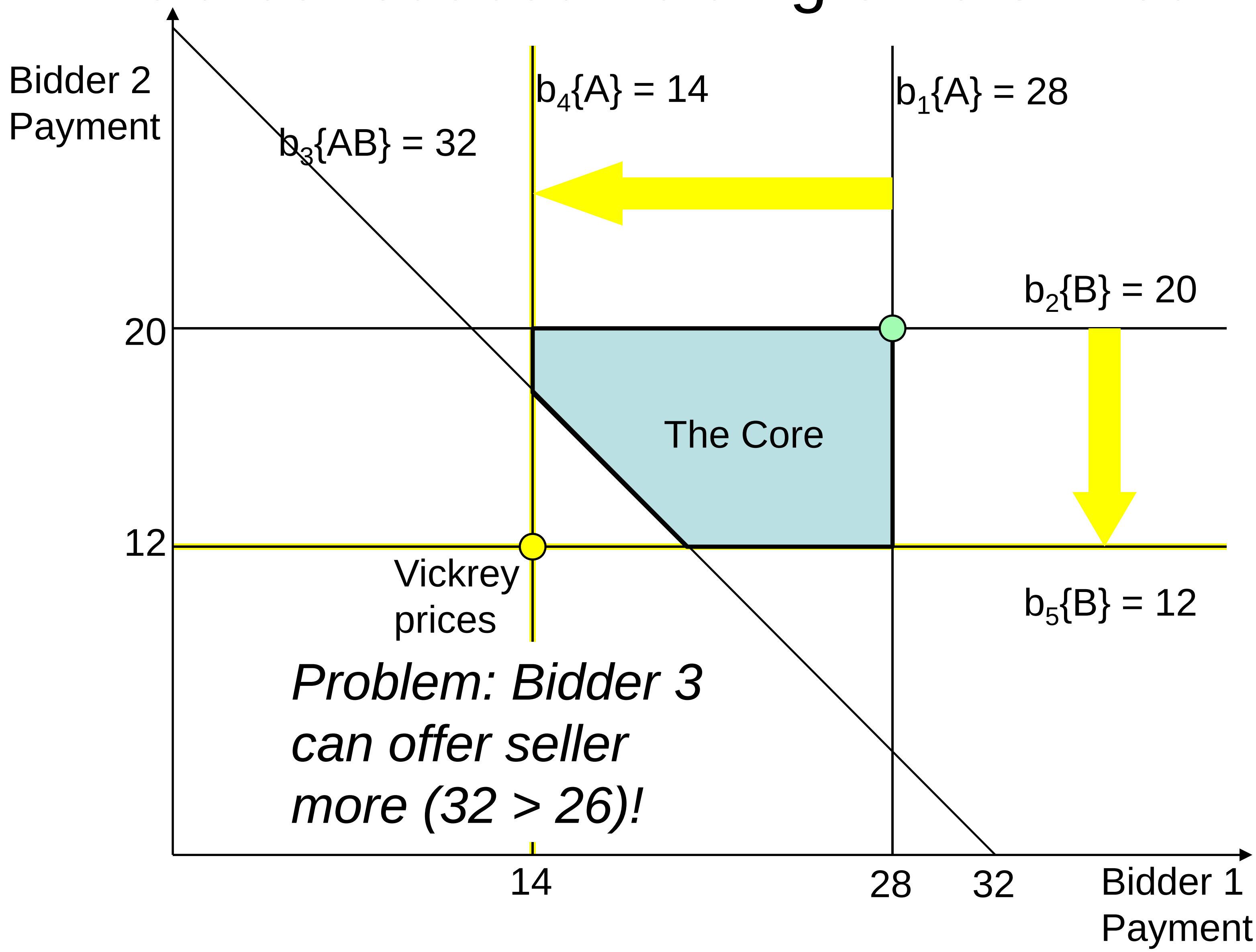
$p_1 = 14$

$p_2 = 12$

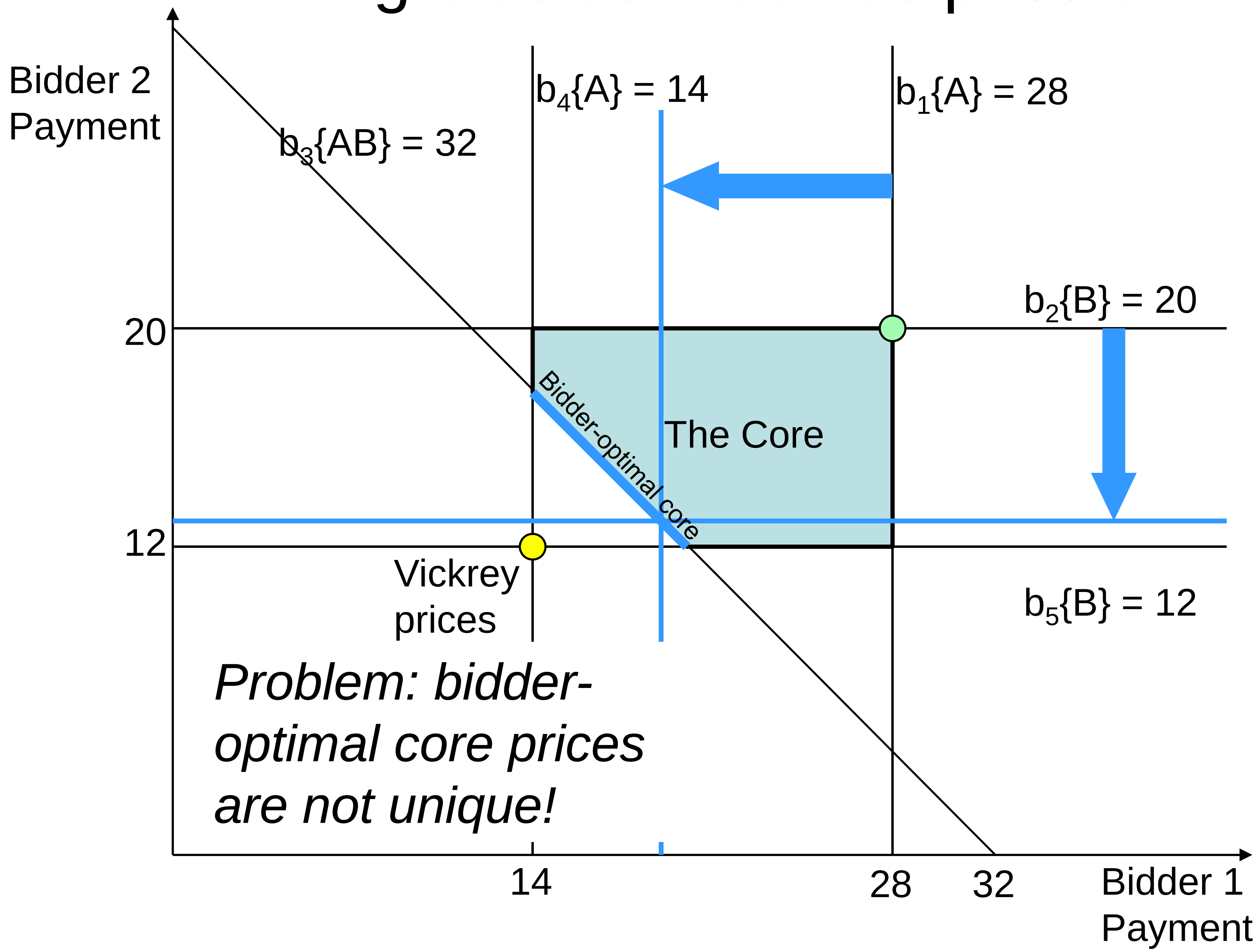
The Core


Bidder 2 Payment
$b_4\{A\} = 14$
$b_1\{A\} = 28$
$b_3\{AB\} = 32$
Efficient outcome
20
$b_2\{B\} = 20$
The Core
12
$b_5\{B\} = 12$
14
28
32
Bidder 1 Payment

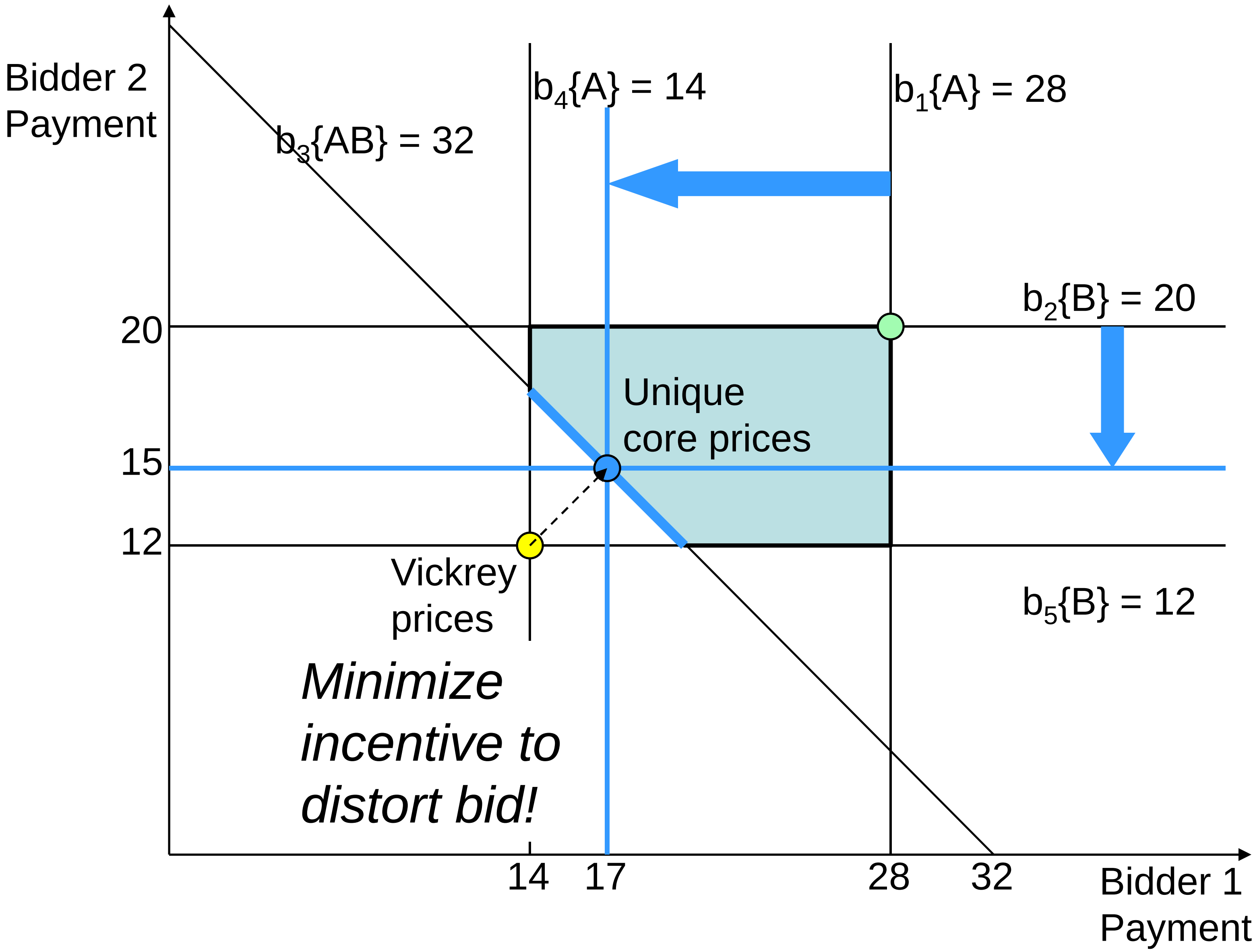
Vickrey prices: How much can each winner's bid be reduced holding others fixed?



Bidder-optimal core prices: *Jointly* reduce winning bids as much as possible



Core point closest to Vickrey prices



Why core pricing?

- Truthful bidding nearly optimal
  - Simplifies bidding
  - Improves efficiency
- Same as Vickrey if Vickrey in core (substitutes)
- Avoids Vickrey problems with complements
  - Prices that are too low
- Revenue is monotonic in bids and bidders
- Minimizes incentive to distort bids

Activity rule

Activity rule: Eligibility points

- Clock stage: Cannot increase package size
- Supplementary bids: Whenever reduce package size, value on all larger packages capped by prices at time of reduction
  - Example
    - Bidder drops from package of size 10 to 6 at prices p
    - For all packages q of size 7 to 10, bid $\leq q \cdot p$
- Implication
  - Profit maximization is poor strategy
  - Bid to maximize package size subject to profit ≥ 0

Full-scale test of design (Maryland and GMU PhD students)

- Experienced subjects
  - PhD course in game theory and auctions
  - Prior participation in package clock auction
- Motivated subjects
  - Average subject payment = $420
- Realistic scenarios

Result

- Activity rule causes major deviation from straightforward bidding
  - Undermines price discovery
  - Reduces efficiency

Activity rule readily fixed: Revealed preference

- At time $t' > t$, package $q_{t'}$ has become relatively cheaper than q_t

  (P′) $q_{t'}\cdot(p_{t'} - p_t) \le q_t\cdot(p_{t'} - p_t)$
- Supplementary bid $b(q)$ must be less profitable than revised package bid at t

  (S′) $b(q) \le b(q_t) + (q - q_t)\cdot p_t$

Example

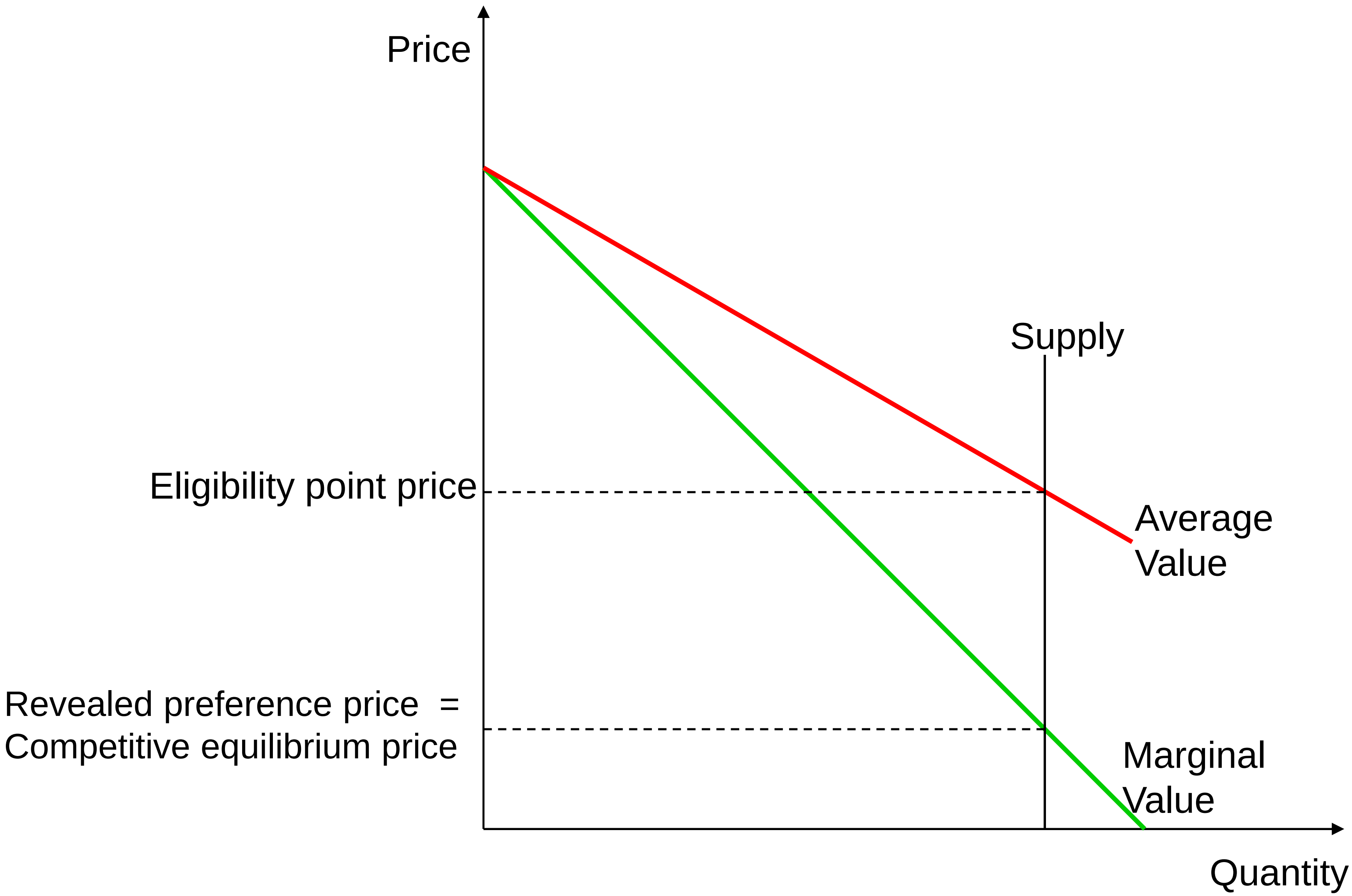
- Revealed preference
  - Bid on most profitable package (max profit)
  - Move up marginal value (demand) curve
- Eligibility point
  - Bid on largest profitable package (max size)
  - Move up average value curve

	Marginal Value		Average Value	
	Bidder A	Bidder B	Bidder A	Bidder B
1 lot	16	8	16	8
2 lots	2	2	9	5

Each wins one; price = 2
Competitive equilibrium!

A wins both; price = 8
Too concentrated; too high priced!

Aggregate demand downward sloping
$\Rightarrow$ Average value > marginal value



Example with constant elasticity

Weaker bidder reveals too much

	Demand Elasticity				
	0.50	0.60	0.70	0.80	0.90

	Marginal Value					Average Value				
Lots	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E	Bidder A	Bidder B	Bidder C	Bidder D	Bidder E
1	10,000	4,642	2,683	1,778	1,292	10,000	4,642	2,683	1,778	1,292
2	2,500	1,462	997	748	598	6,250	3,052	1,840	1,263	945
3	1,111	744	558	450	381	4,537	2,282	1,413	992	757
4	625	461	370	314	277	3,559	1,827	1,152	823	637
5	400	317	269	238	216	2,927	1,525	975	706	553
6	278	234	207	189	176	2,486	1,310	847	620	490
7	204	181	166	156	149	2,160	1,149	750	553	441
8	156	145	138	132	128	1,909	1,023	674	501	402

Too concentrated

Bidding norm	Clearing Price
Max profit	370
Max size	1,292
Difference	249%

Price too high

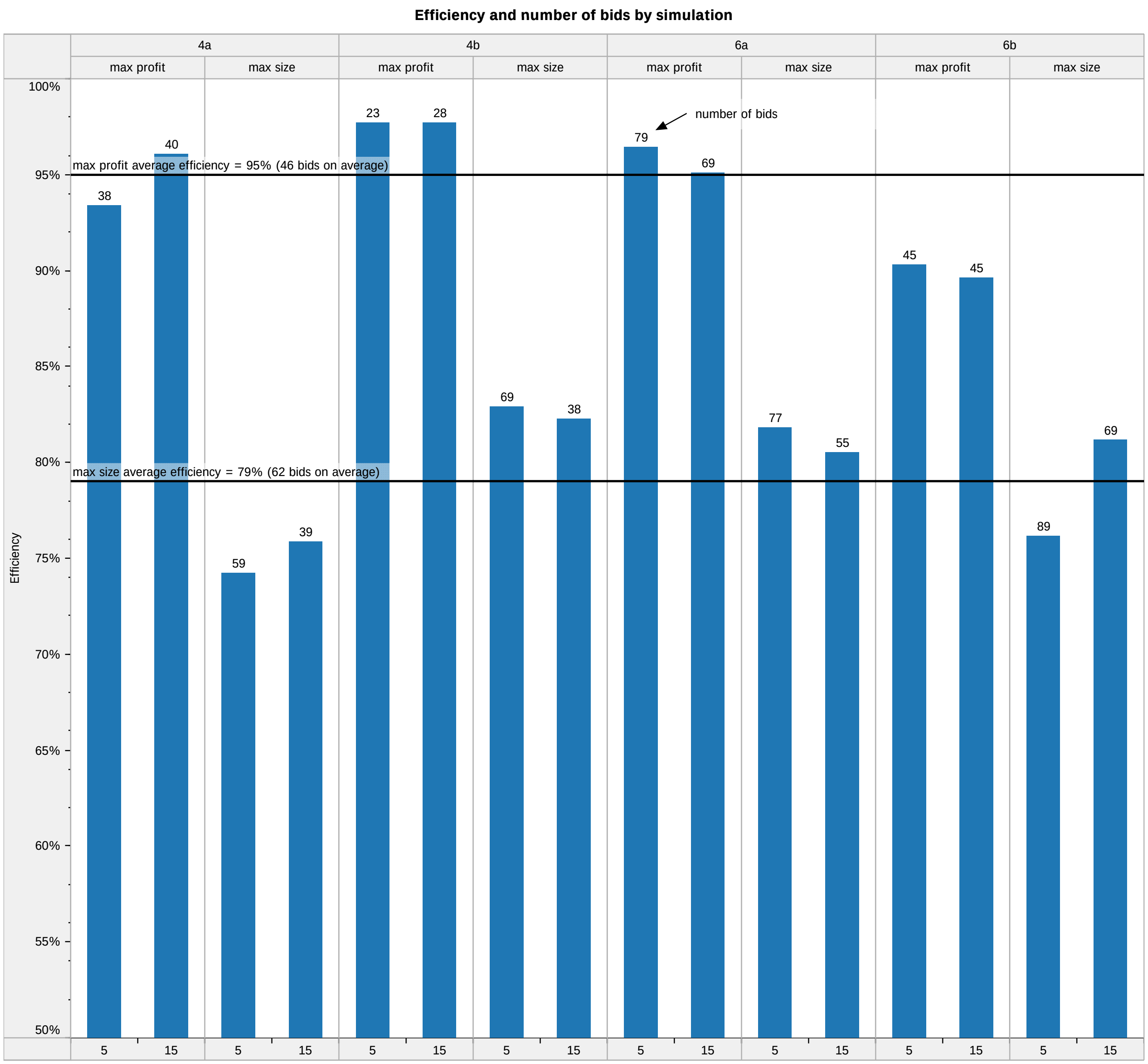
Supply	18
Misassigned	5
Fraction misassigned	28%

	Total Value
Max profit	31,428
Max size	29,150
Inefficiency	7.3%

Comparison of activity rules

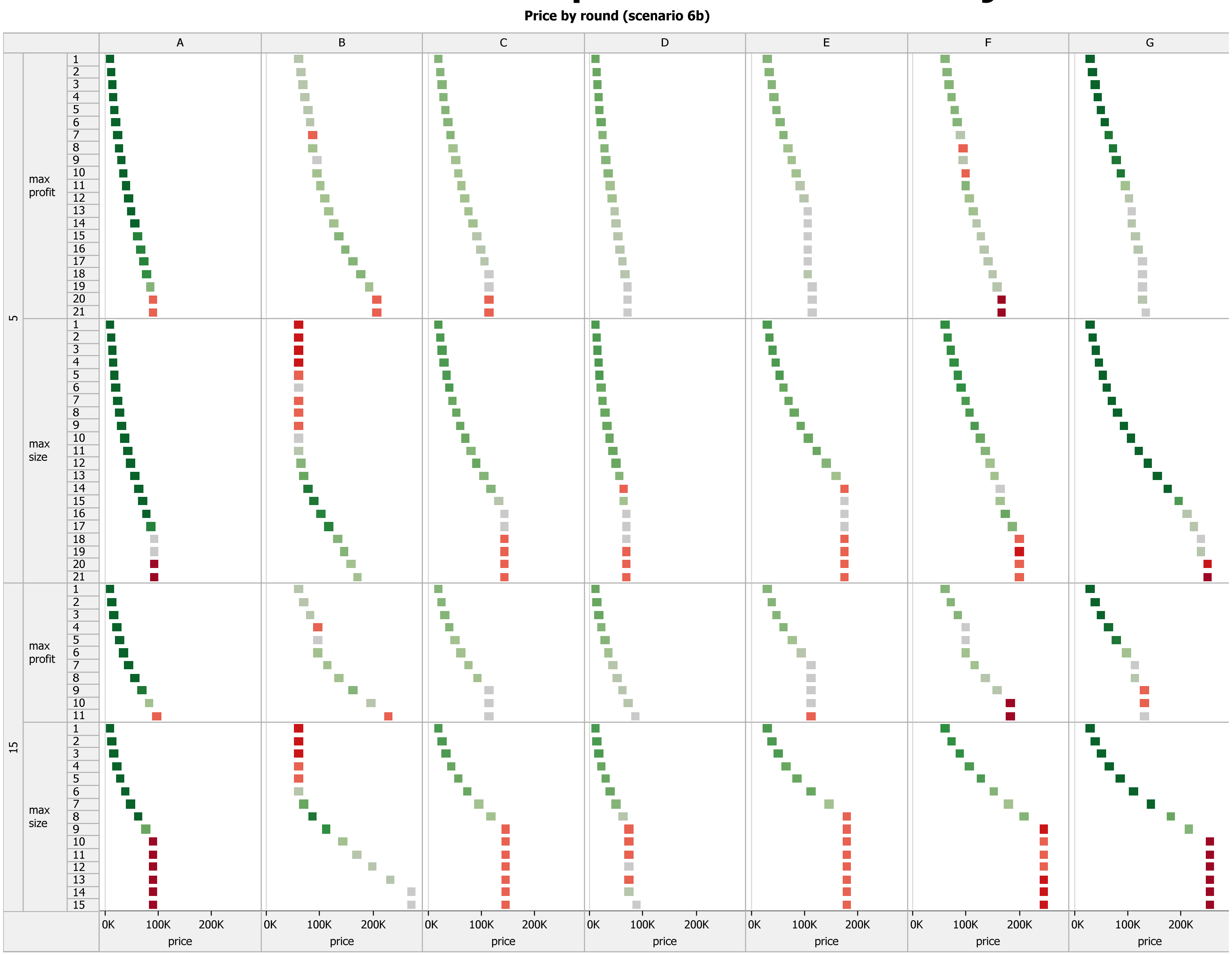
- Eligibility points
  - Bid on largest profitable package
- Revealed preference
  - Bid on profit maximizing package
- Hypothesis
  - Profit maximization yields much better price discovery
- Simulate clock auction under each bidding norm to test hypothesis

RP: Higher efficiency from fewer bids



5 = low bid increments (5 to 15%); 15 = high bid increments (15 to 30%).

RP: Better price discovery



Price for each round by category. Color shows excess demand. Top two panels have small increments (5% to 15%); bottom two panels have large increments (15% to 30%). Under "profit" case, bidders select the profit maximizing package; under "size" bidders select the size maximizing package subject to a positive profit constraint.

Summary of comparison

- Revealed preference, compared with eligibility point rule, yields
  - Substantially higher auction efficiency
  - About the same revenue
  - Substantially higher bidder profits
  - More winners, less concentration
  - Better price discovery with final clock prices closer to competitive equilibrium levels

Problem with revealed preference

- Bidders values change over auction as a result of common value uncertainty
- Revealed preference is complex in supplementary round
  - Single bid can violate many constraints
  - Difficult to see how best to resolve violations

Simplified revealed preference: Include only a few RP constraints

- Clock stage
  - Can always shift to smaller packages
  - Can shift to a larger package that has become relatively cheaper
- Supplementary bids
  - Packages q not larger than the final clock package q_f are capped by revealed preference with respect to q_f

$$b(q) \leq b(q_f) + (q - q_f) \cdot p_f$$

  - Packages q larger than q_f are capped by revealed preference with respect to next smaller package q_s

$$b(q) \leq b(q_s) + (q - q_s) \cdot p_s$$

Properties with substitutes

- Bidding on most profitable package is best
- Clock yields competitive equilibrium with efficient assignment and supporting prices
- No supplementary bids needed
- Final assignment = clock assignment
- Prices reduced to opportunity costs

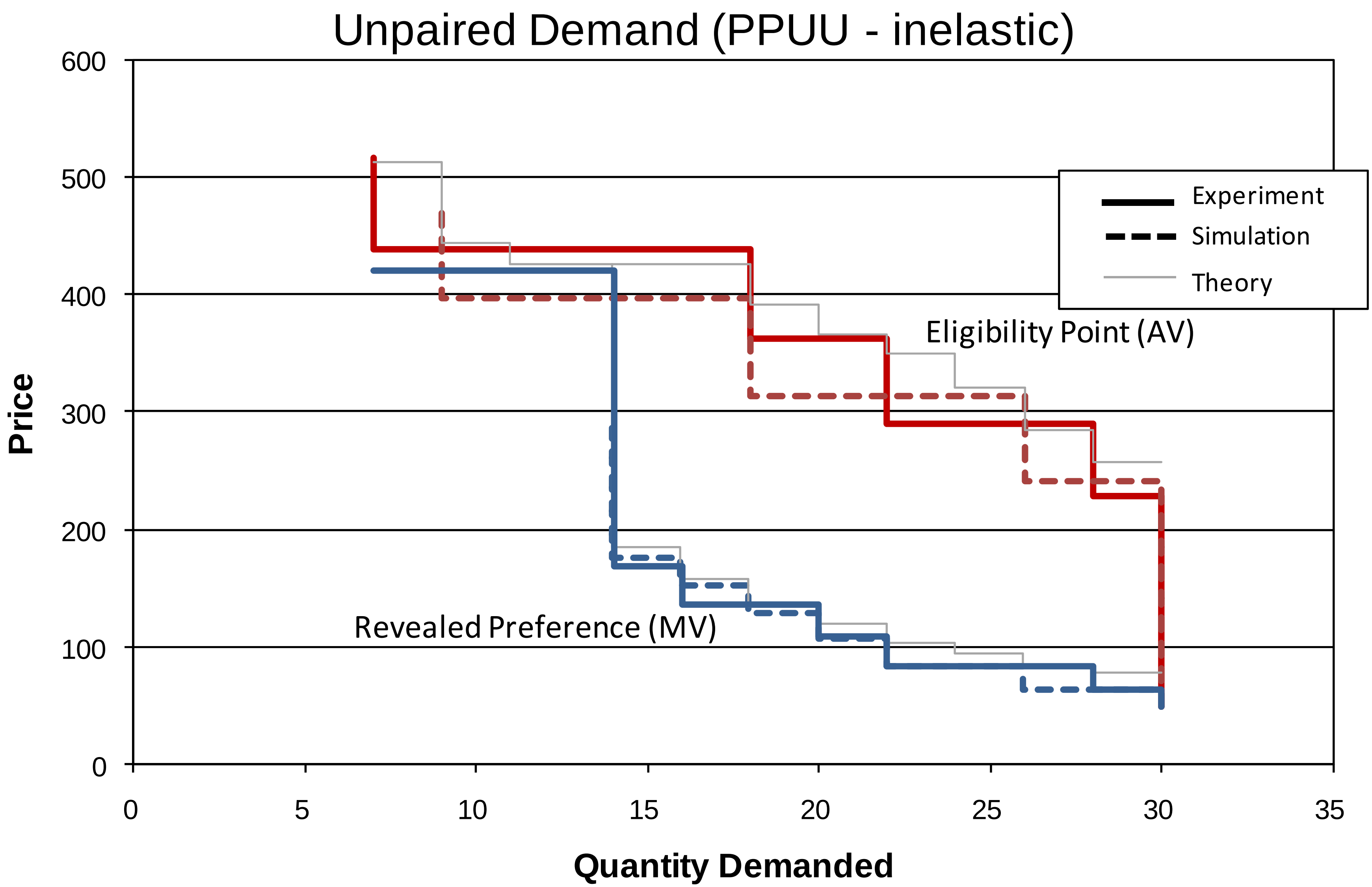
Properties in general

- Bidding on most profitable package is nearly best
- If no unsold lots at end of clock, then
  - Final assignment = clock assignment
  - No supplementary bids needed
- If unsold lots at end of clock, then
  - Supplementary bids needed
  - Clock winner can guarantee it wins final clock package
    (raise by final clock price of unsold lots)

Experimental results

- 100% efficiency in nearly all cases
- Safe strategy adopted by bidders
  - Clock stage
    - RP: Bid on most profitable package
    - EP: Bid on largest profitable package
  - Supplementary round
    - Bid full value on all relevant packages
  - Assignment stage
    - Bid incremental value for specific assignments

Model, simulation, and experiment



Conclusion

Conclusion

- Package clock auction
  - Eliminates exposure
  - Eliminates gaming
  - Enhances substitution
  - Allows auction to determine band plan
  - Readily customized to a variety of settings
  - Many other applications (airport slot auctions)

Conclusion

- Harness power of markets
- Improve pricing
  - Efficient decisions, short term and long term
  - Innovation from price incentives
- Enhance competition
  - Price transparency
  - Enhanced substitution and liquidity
  - Reduced transactions costs
- Mitigate market failures
  - Market power, coordination, externalities, …