A Practical Combinatorial Auction: The Clock-Proxy Auction*

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Abstract

We propose the clock-proxy auction as a practical means for auctioning many interdependent items. A clock auction is followed by a last-and-final proxy round. The approach combines the simple and transparent price discovery of the clock auction with the efficiency of the proxy auction. Linear pricing is maintained as long as possible, but then is abandoned in the proxy round to improve efficiency and enhance seller revenues. The approach has many advantages over the simultaneous ascending auction. In particular, the clock-proxy auction has no exposure problem, eliminates incentives for demand reduction, and prevents most collusive bidding strategies.

1 Introduction

In this chapter we present a promising method for auctioning many interdependent items. A typical application is a spectrum sale in which some licenses are substitutes and some are compliments. The approach combines two auction formats—the clock auction and the proxy auction—to produce an auction with the benefits of both.

In the clock auction, the auctioneer announces prices, one for each of the items being sold. The bidders then express the quantity of each item desired at the current prices. Prices for items with excess demand then increase, and the bidders again express quantities at the new prices. This process is repeated until there are no items with excess demand.

The proxy auction is a particular package bidding procedure with desirable properties (see Ausubel and Milgrom 2002 and chapter 4). The bidders submit their values to computer proxy agents. The proxy agents iteratively submit package bids, selecting the best profit opportunity given the inputted values. The auctioneer then selects the provisionally-winning bids that maximize revenues. This process continues until the proxy agents have no new bids to submit.

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The clock-proxy auction begins with a clock phase and ends with a final proxy round. First, bidders directly submit bids in a simultaneous clock auction, until there is no excess demand on any item. Then bidders have a single opportunity to input proxy values. The proxy round concludes the auction. All bids are kept live throughout the auction. There are no bid withdrawals. The bids of a particular bidder are mutually exclusive. There is an activity rule throughout the clock phase and between the clock phase and the proxy round.

The clock-proxy auction has many advantages. The clock phase is simple for bidders and provides essential price discovery. Porter et al. (2003) demonstrate the clock auction’s success in laboratory experiments, and Ausubel and Cramton (2004) discuss its success in the field. The proxy phase yields an efficient assignment and competitive revenues, while minimizing the opportunities for collusion.

An important benchmark of comparison is the simultaneous ascending auction (see chapter 5 and Milgrom 2004). This auction form performs well when items are substitutes and competition is strong. The clock-proxy auction also does well in this simple setting. In particular, when items are substitutes and market power is not a problem, then the outcome is largely determined in the clock phase and should be similar to the outcome of a simultaneous ascending auction. However, the clock-proxy auction handles complications, like complements, collusion, and market power, much better than the simultaneous ascending auction. The clock-proxy auction should perform better than the simultaneous ascending auction both on efficiency and revenue grounds outside of the simplest settings.

We begin by motivating and describing the clock phase. Then we examine the proxy phase. Finally we put the two together in the clock-proxy auction, describing the important role played by both phases, comparing the auction with the simultaneous ascending auction, and discussing implementation issues.

2 Clock phase

The simultaneous clock auction is the practical implementation of the fictitious “Walrasian auctioneer.” The auctioneer announces prices. The bidders respond with quantities desired at the specified prices. Then the prices are adjusted according to excess demand. And the process is repeated until there is no excess demand for any item.
The clock phase has two important benefits. First, it is simple for the bidders. At each round, the bidder simply expresses the quantities desired at the current prices. Item prices mean that it is trivial to evaluate the cost of any package—just the sum of the item prices. Much strategizing is removed by limiting the bidders’ information to a reporting of the excess demand for each item. Complex bid signaling and collusive strategies are eliminated, since the bidders cannot see individual bids, but only aggregate information. Second, the clock phase has highly-useable price discovery, because of the item prices (linear pricing). With each bidding round, the bidders get a better understanding of relevant prices. This is essential information in guiding the bidders’ decision making. Bidders are able to focus their valuation efforts on the most relevant portion of the price space. As a result, the valuation efforts are more productive. Bidder participation costs fall and efficiency improves.

The weakness of the clock auction is its use of linear pricing at the end of the auction. This means that, to the extent that there is market power, bidders will have an incentive to engage in demand reduction to favorably impact prices. This demand reduction implies that the auction outcome will not be fully efficient (Ausubel and Cramton 2002). However, this inefficiency will be eliminated by the proxy phase.

There are several design choices that will improve the performance of the clock phase. Good choices can avoid the exposure problem, improve price discovery, and handle discrete rounds.

2.1 Avoiding the exposure problem

One important issue in clock auctions is how to treat bids that if accepted would make aggregate demand less than supply. For example, for a particular item, demand may equal supply, so the price of the item does not increase, but the price of a complementary item may increase. The most common approach is to prevent quantity reductions on items for which the price does not increase, but this creates an exposure problem when some items are complements. A better approach in our context is to allow the reduction. A bidder can drop quantity for any item so long as the price has increased on some item the bidder is purchasing. This rule eliminates the exposure problem. The bidder is given the flexibility to drop quantity on items for which there is no price increase.

Another case arises when for a particular item, demand is greater than supply, so the price increases, and one or more bidders attempt to reduce their demands, making demand less than
supply. The common approach in this case is to ration the bidders’ reductions so that supply equals demand. However, this creates an exposure problem when some items are complements. Our approach is not to ration the bidders. All reductions are accepted in full.

The reason for the common restrictions on quantity reductions is to avoid undersell (ending the auction at a point where demand is less than supply). However, these restrictions create an exposure problem. Bidders may be forced to purchase quantities that do not make sense given the final price vector. We eliminate these restrictions and avoid the exposure problem. The consequence is the possibility of undersell in the clock phase, but this is of little importance, since the proxy round can resolve any undersell.

We have conducted nearly a dozen high-stake clock auctions using this rule for electricity products, some of which are substitutes and some of which are complements. These are clock-only auctions without a proxy round. However, since the auctions are conducted quarterly, any undersell in the current auction is added to the quantities in the next auction. Our experience has been that undersell typically is slight (only a few percent of the total). The one exception was an auction in which there was a large negative market price shock near the end of the auction, which resulted in undersell of about fifty percent.

With our rule the clock auction becomes a package auction. For each price vector, the bidder is expressing the package of items desired.

All bids in the clock phase are kept live in the proxy round. Including these bids has two effects. It potentially increases revenues after the proxy phase by expanding choices in the winner determination problem. And it encourages sincere bidding in the clock phase, since bidders are on the hook for all earlier bids. For example, a bidder may decide to bid on items it does not want, but suspects a competitor does want. It does this to increase the competitor’s price and tighten the competitor’s budget constraint. However, such a strategy is risky when all bids are kept live. The strategic bidder may find he has bought something that he does not want.

2.2 Improving price discovery

Price discovery is undermined to the extent that bidders do not bid in a way that is consistent with their true preferences. One possibility is that bidders will choose to underbid in the clock phase, hiding as a “snake in the grass” to conceal their true interests from their opponents. To limit this form of insincere bidding, clock auctions and simultaneous ascending auctions
typically have an activity rule. A bidder desiring large quantities at the end of the auction must bid for quantities at least as large early in the auction, when prices are lower.

Some clock auctions have performed well in the laboratory without any activity rule (Porter et al. 2003). We suspect that this is because of the limited information that the bidders have about the preferences of the others. This lack of information makes it difficult for participants to know how best to deviate from the straightforward strategy of bidding to maximize profits, ignoring one’s impact on prices. In practice, activity rules appear to be important, because of the more detailed knowledge bidders have about the preferences of others and hence a better sense of the benefits of deviating from straightforward bidding. The first U.S. broadband auction is a good example of an auction where the activity rule played an important role (McAfee and McMillan 1996; Cramton 1997).

The most common activity rule in clock auctions is monotonicity in quantity. As prices rise, quantities cannot increase. Bidders must bid in a way that is consistent with a (weakly) downward sloping demand curve. This works well when auctioning identical items, but is overly restrictive when there are many different products. If the products are substitutes, it is natural for a bidder to want to shift quantity from one product to another as prices change, effectively arbitraging the price differences between substitute products. A weaker activity requirement is a monotonicity of aggregate quantity across a group of products. This allows full flexibility in shifting quantity among products in the group. This is the basis for the FCC’s activity rule.

A weakness of the rule based on monotonicity of aggregate quantities is that it assumes that quantities are readily comparable. For example, in the FCC auctions, the quantity associated with a license is the bandwidth of the license times the population covered (MHz-pop). If prices on a per MHz-pop basis vary widely across licenses, bidders may have an incentive to bid on cheap licenses to satisfy the activity rule. This distortion in bidding compromises price discovery.

We propose an alternative activity rule based on revealed preference. The rule depends on both prices and quantities. It is derived from standard consumer theory. Consider two times \( s \) and \( t \) \((s < t)\). Let \( p^s \) and \( p^t \) be the price vectors at these times, let \( x^s \) and \( x^t \) be the associated bidder demands, and let \( v(x) \) be the value of the package \( x \). Revealed preference says that the bidder prefers \( x^s \) to \( x^t \) when prices are \( p^s \):

\[
v(x^s) - p^s \cdot x^s \geq v(x^t) - p^s \cdot x^t
\]
and prefers $x'$ to $x^*$ when prices are $p'$:

$$v(x') - p'x' \geq v(x^*) - p'x^*.$$

Adding these two inequalities yields the revealed preference activity rule:

(RP) \hspace{1cm} (p' - p^s)(x' - x^*) \leq 0.

At every time $t$, the bidder’s demand $x'$ must satisfy (RP) for all times $s < t$.

For the case of a single good, (RP) is equivalent to the condition that as price goes up, quantity cannot increase; that is, bids must be consistent with a weakly downward-sloping demand curve.

Now suppose there are many goods, but all the goods are perfect substitutes in some fixed proportion. For example, the FCC is auctioning 2 MHz licenses and 20 MHz licenses. Ten 2 MHz blocks substitute perfectly for one 20 MHz block. In this simple case, we would want (RP) to do the same thing it does when the perfect substitutes are auctioned as a single good. Indeed, this is the case.

First suppose that all prices are consistent with the rate of substitution (e.g., the 20 MHz block is 10 times as expensive as the 2 MHz block) and all are increasing by the same percentage. The bidder then only cares about the total quantity in MHz and does not care about which goods are purchased. In this case, (RP) allows the bidder to substitute arbitrarily across goods. (RP) is satisfied with equality so long as the bidder maintains the same total MHz in response to the higher prices, and inequality if the bidder reduces total MHz.

Second suppose that the prices are not consistent with the rate of substitution. Say the price on the 2 MHz block increases too fast relative to the 20 MHz block. The bidder then wants to shift all its quantity to the 20 MHz block, and (RP) allows this: since the 20 MHz is relatively cheaper, (RP) gives the bidder more credit for dropping quantity on the 2 MHz blocks than the bidder is debited for the increase in the 20 MHz block. It might seem that the mispricing allows the bidder to expand quantity somewhat, but this is not the case. Since (RP) is required with respect to all previous bids, the bidder would be constrained by its maximum quantity the last time the 20 MHz block was the best value.

We conclude that (RP) does just the right thing in the case of many perfect substitutes. The activity rule is neither strengthened nor weakened by alternative product definitions.
Now suppose some goods are perfect complements in fixed proportion. For example, in an electricity auction, the bidder wants to maintain a 2 to 1 split between baseload product and peakload product. If there are just these two products, then the bidder just cares about the weighted sum of the product prices. As prices increase, the bidder certainly satisfies (RP) by maintaining the same quantities or by reducing the quantities in the desired ratio; however, the bidder is unable to increase quantities. (RP) does just the right thing in the case of perfect compliments.

If we combine the two cases above so that some goods are perfect substitutes and some are perfect complements. Then (RP) still does the right thing. Bidders will want to shift quantity to the cheapest substitute in building the package of complements. Shifting away from substitute products for which price is increasing too quickly yields a credit that exceeds the debit from shifting toward the relatively cheap product. Hence, this is allowed under (RP). Moreover, (RP) prevents a bidder who has bid straightforwardly (in the sense of bidding on the cheapest substitutes) from expanding its quantity of complementary packages as prices rise.

It is useful to compare (RP) with the current FCC activity rule, which ignores prices and simply looks at aggregate quantity in MHz-pop. Parking is the main problem created by the current rule. To maintain flexibility, a bidder has an incentive to bid on underpriced markets or low-value markets with high quantity. The bidder does this for two reasons: (1) to keep the prices on desired products from increasing too quickly, while maintaining the flexibility to expand demand on products for which competitor demands fall off faster than expected, and (2) to maintain the flexibility to punish a rival by shifting quantity to the rival’s desired markets if the rival does not back out of the bidder’s desired markets. Thus, parking is motivated by demand reduction and tacit collusion. But in the clock implementation, collusion is mitigated, because bidders see only excess demand; they do not have the information to know when retaliation is needed, where the retaliation should occur, or how to avoid retaliation. And demand reduction is mitigated by the final proxy round. Hence, we should expect parking to be much less of a problem in the clock implementation.

The greatest damage from parking comes from price distortions that exclude the efficient winner in favor of the bidder parking. Under the FCC rule, bidders get the greatest parking value from low-price high-quantity licenses. These may get bid up to the point where the efficient winner drops out. Note that the parking value is retained even when there is no excess demand
for the high-quantity license. The parking bidder can withdraw from the license freeing up a large quantity of eligibility. In contrast, the (RP) rule only gives eligibility credit to the extent that the good is underpriced. That is, the credit for dropping quantity on a good without excess demand (so the price does not increase) is zero. Parking is only effective when bidding on underpriced goods. But parking on underpriced goods does no harm. The parking simply serves to increase the price of the underpriced good. Hence, it appears that the revealed-preference activity rule may have substantial benefits over the current FCC activity rule.

The revealed-preference activity rule admittedly is more complex than the FCC rule based on aggregate quantity. However, it still can be displayed in the same simple way on the bidder’s bid entry screen. As the bid is entered, an activity cell indicates the amount of slack in the tightest (RP) constraint, and changes color when the constraint is violated.

2.3 Handling discrete rounds

Although in theory one can imagine implementing an ascending auction in continuous time, this is hardly ever done in practice. Clock auctions inevitably use discrete rounds for two important reasons. First, communication is rarely so reliable that bidders would be willing to be exposed to a continuous clock. A bidder would find it unsatisfactory if the price clock swept past the bidder’s willingness to pay because of a brief communication glitch. Discrete rounds are robust to communication problems. Discrete rounds have a bidding window of significant duration, rarely less than ten minutes and sometimes more than one hour. This window gives bidders time to correct any communication problems, to resort to back-up systems, or to contact the auctioneer and have the round extended. Second, a discrete round auction may improve price discovery by giving the bidders an opportunity to reflect between rounds. Bidders need time to incorporate information from prior rounds into a revised bidding strategy. This updating is precisely the source of price discovery and its associated benefits.

An important issue in discrete-round auctions is the size of the bid increments. Larger bid increments enable the auction to conclude in fewer rounds, but they potentially introduce inefficiency from the use of a coarse price grid. Large increments also introduce incentives for gaming as a result of the expanded importance of ties. But using small increments especially in an auction with many clocks can greatly increase the number of rounds and, hence, the time required to complete the auction. Bidders generally prefer a shorter auction. A short auction
reduces participation costs. A short auction also reduces exposure to price movements during the auction. This is especially relevant in securities and energy auctions for which there are active secondary markets of close substitutes, and for which underlying price movements could easily exceed the bid increments.

Fortunately it is possible to capture nearly all of the benefits of a continuous auction and still conduct the auction in a limited number of rounds, using the technique of intra-round bids. With intra-round bids, bidders express their demands in each auction round at all prices along the line segment from the start-of-round prices to the end-of-round prices. In a traditional clock auction, price may increase from say $90 to $100 in a round, but the bidder is only able to express the quantity it desires at $90 and at $100. With intra-round bids, the bidder expresses its desired quantity at all prices between $90 and $100. With many products, the bidder expresses quantity changes along the line segment from the start-of-round price vector to the end-of-round price vector. This avoids the inefficiency associated with a coarser price grid. It also avoids the gaming behavior that arises from the increased importance of ties with coarser prices. The only thing that is lost is the within-round price discovery. However, within-round price discovery is much less important than the price discovery that occurs between rounds.

The experience from a number of high-stakes clock auctions indicates that intra-round bidding lets the auctioneer conduct auctions with ten or more products in about ten rounds, with little or no loss from the discreteness of rounds. These auctions can be completed in a single day. By way of contrast, early spectrum auctions and some electricity auctions without intra-round bids took weeks or even months to conclude. In a few instances, the longer duration was warranted due to the enormous uncertainty and extremely high stakes, but generally speaking, intra-round bids would have reduced the bidding costs without any meaningful loss in price discovery.

2.4 End of the clock phase

The clock phase concludes when there is no excess demand on any item. The result of the clock phase is much more than this final assignment and prices. The result includes all packages and associated prices that were bid throughout the clock phase. Due to complementarities, the clock phase may end with substantial excess supply for many items. If this is the case, the final assignment and prices may not provide a good starting point for the proxy phase. Rather bids
from an earlier round may yield an assignment with higher revenue. (When calculating revenues excess supply should be priced at the reserve price, which presumably represents the seller’s opportunity cost of selling the item.)

A sensible approach is to find the revenue maximizing assignment and prices from all the bids in the clock phase. This point is found by backing up the clock to the price point where revenue is at its maximum. The associated linear prices can serve as the lower bound on prices in the proxy phase. That is, for each bidder, the minimum bid on each package is calculated from these revenue maximizing item prices.

In some cases the auctioneer may decide to end the clock phase early—with some excess demand on one or more items. This would be done when the revenue improvements from successive clock rounds are sufficiently small that it is clear that revenues in the clock phase are nearly (or already) maximal. With the proxy phase to follow, there is little loss in either revenues or efficiency from stopping, say when revenue improvements are less than ½ percent for two consecutive rounds. At this point price discovery is largely over on all but the smallest items. Giving the auctioneer the discretion to end the clock phase early also enables the auction to follow a more predictable schedule.

3 Proxy phase

Like the clock auction, the proxy auction is based on package bids. However, the incentives are quite different. The main difference is the absence of linear pricing (item prices). Only packages are priced. This weakens price discovery, but the proxy phase is not about price discovery. It is about providing the incentives for efficient assignment. All the price discovery occurs in the clock phase. The second main difference is that the bidders do not bid directly in the proxy phase. Rather, they submit values to the proxy agents, who then bid on their behalf using a specific bidding rule. The proxy agents bid straightforwardly to maximize profits. The proxy phase is a last-and-final opportunity to bid.

The proxy auction works as follows (see Ausubel and Milgrom 2002 and chapter 4). Each bidder reports his values to a proxy agent for all packages that the bidder is interested in. Budget constraints can also be reported. The proxy agent then bids in an ascending package auction on behalf of the real bidder, iteratively submitting the allowable bid that, if accepted, would maximize the real bidder’s profit (value minus price), based on the reported values. The auction
is conducted with negligibly small bid increments. After each round, provisionally winning bids are determined that maximize seller revenue from compatible bids. All of a bidder’s bids are kept live throughout the auction and are mutually exclusive. The auction ends after a round with no new bids.

The advantage of this format is that it ends at a point in the core for the reported preferences. The coalition form game \((L, w)\) where \(L\) denotes the set of players \((l = 0\) is the seller and the rest are the bidders\) and \(w(S)\) denotes the value of coalition \(S\). If \(S\) excludes the seller, then \(w(S) = 0\); if \(S\) includes the seller, then

\[
    w(S) = \max_{x \in X} \sum_{i \in S} v_i(x_i).
\]

The \(Core(L, w)\) is the set of all profit allocations that are feasible for the coalition of the whole and cannot be blocked by any coalition \(S\); that is, each coalition \(S\) gets at least \(w(S)\).

**Theorem** (Ausubel and Milgrom 2002). *The outcome of the proxy auction is a point in the Core\((L, w)\) relative to the reported preferences.*

The core outcome assures competitive revenues for the seller. The seller cannot do better by excluding one or more bidders. Moreover, the core outcome assures allocative efficiency. The proxy auction is not subject to the inefficiency of demand reduction.

If the items are substitutes, then the Vickrey payoff is the bidder-Pareto-optimal point in the core, and the outcome of the proxy auction coincides with the outcome of the Vickrey auction. If the goods are not substitutes, then the Vickrey payoff is not in the core and the proxy auction yields an outcome with higher seller revenues.

**Theorem** (Ausubel and Milgrom 2002). *If \(\pi\) is a bidder-Pareto-optimal point in the Core\((L, w)\), then there exists a Nash equilibrium of the proxy auction with associated payoff vector \(\pi\).*

This is a complete information result. These equilibria may be obtained using strategies of the form: bid your true value minus a nonnegative constant on every package.

The reason that the Vickrey auction does not coincide with the core when goods are not substitutes is that adding a bidder to the auction can lower seller revenue. For example, suppose there are two identical items and three bidders. Bidder 1 values the pair only at $2. Bidder 2 and 3 each value a single item only at $2. The Vickrey auction awards one item each to bidders 2 and
3. The Vickrey auction gives each bidder its incremental contribution. The social value with bidder 2 is $4 and without bidder 2 is $2. Hence, bidder 2 gets a profit of $2 and pays $0. The same holds for bidder 3. The seller gets $0. Adding bidder 3 reduced Vickrey revenues from $2 to $0, which is why the Vickrey outcome is outside the core. In contrast, the proxy auction yields revenues of $2. Adding a bidder in a proxy auction can never reduce seller revenues.

4 The clock-proxy auction

The clock-proxy auction begins with a clock auction for price discovery and concludes with the proxy auction to promote efficiency.

The clock auction is conducted with the revealed-preference activity rule until there is no excess demand on any item. The market-clearing item prices determine the initial minimum bids for all packages for all bidders. Bidders then submit values to proxy agents, who bid to maximize profits, subject to a relaxed revealed-preference activity rule. Bids from the clock phase are kept live as package bids in the proxy phase. All of a bidder’s bids, both clock and proxy, are mutually exclusive. The auctioneer selects the provisionally-winning bids as those that maximize seller revenue.

4.1 Relaxed revealed-preference activity rule

To promote price discovery in the clock phase, the proxy agent’s allowable bids must be constrained by the bidder’s bids in the clock phase. The constraint we propose is a relaxed version of the revealed preference activity rule.

First, we restate revealed preference in terms of packages and the associated minimum bids for the packages. Consider two times $s$ and $t$ ($s < t$). Suppose the bidder bids for the package $S$ at time $s$ and $T$ at time $t$. Let $P^s(S)$ and $P^s(T)$ be the package price of $S$ and $T$ at time $s$; let $P^t(S)$ and $P^t(T)$ be the package price of $S$ and $T$ at time $t$; and let $v(S)$ and $v(T)$ be the value of package $S$ and $T$. Revealed preference says that the bidder prefers $S$ to $T$ at time $s$:

$$v(S) - P^s(S) \geq v(T) - P^s(T)$$

and prefers $T$ to $S$ at time $t$:

$$v(T) - P^t(T) \geq v(S) - P^t(S).$$

Adding these two inequalities yields the revealed preference activity rule for packages:
\( (RP') \quad P^s(S) - P^s(S) \geq P^t(T) - P^t(T). \)

Intuitively, the package price of \( S \) must have increased more than the package price of \( T \) from time \( s \) to time \( t \), for otherwise, at time \( t \), \( S \) would be more profitable than \( T \).

Notice that the constraint \( (RP') \) is automatically satisfied at any two times in the proxy phase, because the proxy agent is required to bid to maximize profits. However, an activity rule based on \( (RP') \) is too strict when comparing a time \( s \) in the clock phase with a time \( t \) in the proxy phase. Due to the linear pricing in the clock phase, the bidders have an incentive to reduce demands below their true demands. One purpose of the proxy phase is to let the bidders undo any inefficient demand reduction that has taken place in the clock phase and to defect from any collusive split of the items. Hence, it is important to let the bidders expand their demands in the proxy phase. The amount of expansion required depends on the competitiveness of the auction.

We propose a relaxed revealed-preference activity rule:

\( (RRP) \quad \alpha[P^s(S) - P^s(S)] \geq P^t(T) - P^t(T). \)

At every time \( t \) in the proxy phase, the bidder can bid on the package \( T \) only if \( (RRP) \) is satisfied for every package \( S \) bid at time \( s \) in the clock phase.

The parameter \( \alpha > 1 \) is chosen by the auctioneer based on the competitiveness of the auction. For highly competitive auctions little demand reduction is likely to occur in the clock phase and \( \alpha \) can be set close to 1. On the other hand, if there is little competition (and high concentration) then \( \alpha = 2 \) may be appropriate.

### 4.2 Why include the clock phase?

The clock phase provides price discovery that is essential to an efficient auction process. The feedback of linear prices is extremely useful to the bidders. At each round bidders are faced with the relatively simple problem of expressing demands at specified prices. Moreover, since there is no exposure problem, bidders can bid for synergistic gains without fear. Prices then adjust in response to excess demand. As the bidding continues, bidders get a better understanding of what they may win and where their best opportunities lie.

The case for the clock phase is further strengthened when we recognize that it is costly for bidders to determine their preferences. The clock phase, by providing tentative price information, helps focus the bidder’s decision problem. Rather than consider all possibilities from the outset,
the bidder can instead focus on cases that are important given the tentative price and assignment information. Although this point already is valid in auctions for a single good (Compte and Jehiel 2000), it becomes more critical in the context of many goods, where the bidder’s decision problem is much more complicated. Rather than simply decide whether to buy, the bidder must decide which goods to buy and how many of each. The number of possibilities grows exponentially with the number of goods. Price discovery then plays an essential role in the efficient development of bidder preferences.

Similarly, price discovery in the clock phase makes bidding in the proxy phase vastly simpler. Without the clock phase, bidders would be forced to determine values for all possible packages. The complexity of this task increases exponentially with the number of items being sold. Our experience with dozens of bidders suggests that such an approach is impractical without good price information. With the clock phase, bidders can focus their decision making on the relevant part of the price space. The bidders see that the vast majority of options do not need to be considered—they are excluded by the prices established in the clock phase. The bidders also get a sense of what packages are most promising, and how their demands fit in the aggregate with those of the other bidders.

In competitive auctions where substitution possibilities are strong, we expect the clock phase to do most of the work in establishing prices and assignments—the proxy phase would play a limited role. In these settings, constructing the valuation function for the proxy phase is especially simple. For example, a good approximation may be additive demands at prices above but close to the final clock prices.

When competition is weak it is more likely that the clock phase will end prematurely, but this simply means that the bidder will need to express values over a greater range of prices. Complementarities also complicate the valuation function, but handling them in a much smaller price space is much easier.

4.3 Why include the proxy phase?

The main advantage of the proxy phase is getting an outcome in the core. The proxy phase pushes outcomes toward an efficient allocation, while generating competitive revenues for the seller.
Incentives for demand reduction are eliminated. A large bidder can bid for large quantities without the fear that doing so will adversely impact the price the bidder pays.

The proxy phase also mitigates collusion. Any collusive split of the items established in the clock phase can be undone in the proxy phase. The relaxed activity rule means that the bidders can expand demands in the proxy phase. The allocation is still up for grabs in the proxy phase.

In some sense the clock-proxy auction has similarities with the Anglo-Dutch design initially proposed for the United Kingdom’s third-generation mobile wireless auction (Klemperer 2002). Both formats have an ascending auction followed by a sealed-bid last-and-final round. However, the motivation for the last-and-final round is quite different. In the Anglo-Dutch design, the last round has pay-as-bid pricing intended to introduce inefficiency and hence to motivate inefficient bidders to participate in the auction (and thus raise auction revenues). In the clock-proxy auction, the last round is closer to Vickrey pricing and is intended to promote efficiency, rather than prevent it.

The proxy phase will play a more important role to the extent that competition is limited and complementarities are strong and varied across bidders. Then it is more likely that the clock phase will end prematurely. However, in competitive auctions, the proxy phase may not be needed.

4.4 Comparison with the simultaneous ascending auction

The simultaneous ascending auction as implemented by the FCC is an important benchmark of comparison, given its common use in auctioning many interdependent items (see chapter 5). The clock auction is a variant of the simultaneous ascending auction in which the auctioneer specifies prices and the bidders name quantities. There are several advantages to the clock implementation.

The clock auction is a simpler process than the simultaneous ascending auction. Bidders are provided the minimal information needed for price discovery—the prices and the excess demand. Bidders are not distracted by other information that is largely extraneous.

The clock auction also can take better advantage of substitutes, for example, using a single clock for items that are near perfect substitutes. In spectrum auctions, there is a tendency for the spectrum authority to make specific band plans to facilitate the simultaneous ascending auction. For example, anticipating demands for a large, medium and small license, the authority may
specify a band bland with three blocks—30 MHz, 20 MHz, and 10 MHz. Ideally, these decisions would be left to the market. In a clock auction, the bidders could bid the number of 2 MHz blocks desired at the clock price. Then the auction would determine the band plan, rather than the auction authority. This approach is more efficient and would likely be more competitive, since all bidders are competing for all the bandwidth in the clock auction. With the pre-set band plan, some bidders may be uninterested in particular blocks, such as those that are too large for their needs.

Clock auctions are faster than a simultaneous ascending auction. Simultaneous ascending auctions are especially slow near the end when there is little excess demand. For example, when there are six bidders bidding on five similar licenses, then it typically takes five rounds to get a one bid-increment increase on all items. In contrast, in a clock auction an increment increase takes just a single round. Moreover, intra-round bids allow larger increments, without introducing inefficiencies, since bidders still can express demands along the line segment from the start-of-round prices to the end-of-round prices.

The clock auction limits collusion relative to the simultaneous ascending auction. Signaling how to split up the items is greatly limited. Collusive strategies based on retaliation are not possible, because bidder-specific quantity information is not given. Further, the simultaneous ascending auction can have a tendency to end early when an obvious split is reached, but this cannot happen in the clock auction, since the bidders lack information about the split. Also there are fewer rounds to coordinate a split.

The clock auction (as described here) eliminates the exposure problem. As long as at least one price increases, a bidder can reduce quantity on his other items. The bid is binding only as a full package. Hence, the bidder can safely bid for synergistic gains.

The clock auction mitigates the threshold problem inherent in the simultaneous ascending auction with package bids. Since the clocks are controlled by the auctioneer, there are no jump bids. A large bidder cannot get ahead of its smaller rivals to maximize the threshold problem. Linear pricing in the clock auction means that the small bidder just has to meet the price on an item by item basis.

The clock-proxy auction shares all these advantages of the clock auction, and in addition promotes core outcomes. The proxy phase further mitigates collusion and eliminates demand
reduction. The cost of the proxy phase is added implementation complexity. Also the absence of linear pricing reduces the transparency of the auction. It is less obvious to a bidder why he lost.

### 4.5 Combinatorial exchange

The clock-proxy auction can be extended to handle two-sided auctions. A natural application is the auctioning of encumbered spectrum (Cramton, Kwerel and Williams 1998; Kwerel and Williams 2003). The spectrum authority is selling overlay licenses and incumbents are (potentially) selling their existing rights. The simplest design for this two-sided auction is to use the clock-proxy auction essentially as is. Incumbents simply put their rights on the auction block. The key issue is whether and how the incumbents can participate in the clock-proxy auction, once they decide to turn over their rights. Three options seem reasonable: (1) the incumbents can set an ex ante reserve price, but cannot otherwise bid on their rights, (2) the incumbents can bid on their rights in the proxy phase only, and (3) the incumbents can bid on their rights in both the clock and proxy phase.

**Ex ante reserve.** The incumbent must specify before the bidding starts the minimum price at which he is willing to sell. The incumbent is unable to distort the bidding process, but has an incentive to set too high a reserve. This approach is problematic in situations where the efficient outcome is for the incumbent to buy both his right and the overlay license. The approach puts the incumbent in the same position as the spectrum authority—both are selling subject to a reserve.

**Proxy bidding only.** The incumbent gets the benefit of price discovery in the clock phase before the reserve is set. This may temper the incumbent’s incentive to set too high a reserve, yet it prevents the incumbent from distorting the price discovery in the clock phase.

**Bidding in both the clock and proxy phases.** This approach lets the incumbent act like any other buyer. This makes the most sense in situations where the efficient outcome may well be for the incumbent to retain its right together with the overlay license. The disadvantage is that the incumbent may distort the price discovery in the clock phase.

In the latter two approaches, a potential concern is that the incumbent, through its direct bidding in the auction, will be able to capture a substantial portion of the value of the overlay license. The spectrum authority can limit this problem with two instruments: the reserve price and the increment rule in the clock phase.
4.6 Implementation issues

We briefly discuss three of the most important implementation issues.

Price increments in the clock phase

When auctioning many items, one must take care in defining the price adjustment process. This is especially true when some goods are complements. Intuitively, undersell in the clock phase is minimized by having each product clear at roughly the same time. Otherwise price increases on complementary products can cause quantity drops on products that have already cleared. Thus, the goal should be to come up with a price adjustment process that reflects relative values as well as excess demand. Moreover, the price adjustment process effectively is resolving the threshold problem by specifying who should contribute what as the clock ticks higher. To the extent prices adjust with relative values the resolution of the threshold problem will be more successful.

One simple approach is for the relative value information to be built into the initial starting prices. Then use a percentage increase, based on the extent of excess demand. For example, the percentage increment could vary linearly with the excess demand, subject to a lower and upper limit.

Expression of proxy values

Even with the benefit of the price discovery in the clock phase, expressing a valuation function in the proxy phase is difficult. When many items are being sold, the bidder will need a tool to facilitate translating preferences into proxy values. The best tool will depend on the circumstances.

At a minimum, the tool will allow an additive valuation function. The bidder submits a demand curve for each item. The value of a package is then found by integrating the demand curve (adding the marginal values) up to the quantity of the item in the package, and then adding over all items. This additive model ignores all value interdependencies across items; it assumes that the demand for one item is independent of the demand for other items. Although globally (across a wide range of quantities) this might be a bad assumption, locally (across a narrow range of quantities) this might be a reasonable approximation. Hence, provided the clock phase has taken us close to the equilibrium, so the proxy phase is only doing some fine-tuning of the clock
outcome, then such a simplistic tool may perform reasonably well. And of course it performs very well when bidders actually have additive values.

A simple extension of the additive model allows the bidder to express perfect substitutes and complements within the additive structure. For example, items A and B may be designated perfect complements in the ratio 1 to 3 (one unit of A is needed for three units of B). Then the bidder expresses a demand curve for A and B (with the one to three ratio always maintained). Items C and D may be designated perfect substitutes in the ratio 2 to 1 (two C’s equal one D). Then the bidder expresses a demand curve for C or D (with all quantity converted to C-equivalent). This extension effectively allows the bidder to redefine the items in such a way to make the additive model fit. For example, in a spectrum auction, a bidder for paired spectrum will want to express a demand for paired spectrum. This can be done by designating the upper and lower channels as perfect complements, but then the blocks of paired spectrum as perfect substitutes. A bidder for unpaired spectrum would designate all channels as perfect substitutes, and then express a single demand curve for unpaired spectrum.

Demand curves typically are expressed as step functions, although in some contexts piece-wise linear demand curves are allowed. Bidders should be able to specify whether quantity can be rationed. For example if a bidder drops quantity from 20 to 10 at a price of $5, does this mean the bidder is just as happy getting 14 units as 10 units or 20 units when the price is $5 per unit, or does the bidder only want exactly 10 units at a price of $5, and exactly 20 units at a price of $4.99? Is there a minimum quantity that must be won for the item to have value?

Beyond this, the tool should allow for the inclusion of bidder constraints. Budget constraints are the most common: do not bid more than X. Other constraints may be on quantities: only value A if you win B. This constraint arises in spectrum auctions when a bidder has secondary regions that have value only if the primary regions are won.

The bidders’ business plans are a useful guide to determine how best to structure the valuation tool in a particular application. Business plans are an expression of value to investors. Although the details of the business plans are not available to the auctioneer, a useful valuation tool can be constructed from understanding the basic structure of these business plans.
Calculating prices in the proxy phase

The proxy phase is a sealed-bid auction. At issue is how best to calculate the final assignment and prices. The final assignment is easy. This is just the value maximizing assignment given the reported values. The harder part is determining the prices for each winning package. The clock phase helps by setting a lower bound on the price of each package. Given these starting prices, one approach would be to run directly the proxy auction with negligible bid increments. With many items and bidders this would require voluminous calculations.

Fortunately, the process of calculating prices can be accelerated using various methods (see chapter 21 and Zhong et al. 2003). First, as suggested by David Parkes, package prices for all bidders can start at “safe prices,” defined as the maximum bid on the package by any losing bidder. Second, prices can increase in discrete jumps to the point where a bidder starts bidding on a particular package or stops bidding on a particular package. Although these methods have not yet been fully developed, calculating the prices in the proxy phase likely can be done with many items and bidders in an expedient manner.

The precise process for calculating the prices is especially important when some items are complements, since then there will be a set of bidder-Pareto-optimal points in the core, and the price process will determine which of these points is selected.

5 Conclusion

We propose the clock-proxy auction for auctioning many interdependent items—a simultaneous clock auction followed by a last-and-final proxy round. The basic idea is to use linear prices as long as possible to maximize price discovery, simplicity, and transparency. The clock phase also greatly facilitates the bidders’ valuation analysis for the proxy round, since the analysis can be confined to the relevant part of the price space identified in the clock phase. Finally, unlike the simultaneous ascending auction, the clock auction does not suffer from the exposure problem.

For competitive auctions of items that are mostly substitutes, the clock auction without the proxy round will perform well. Indeed a clock auction without a proxy round may be the best approach in this setting, since it offers the greatest simplicity and transparency, while being highly efficient.
With limited competition or items with a complex and varied structure of complements, adding the proxy phase can improve auction outcome. In particular, a core outcome is achieved. Seller revenues are competitive and the allocation is efficient. The demand reduction incentive present in the clock phase is eliminated. Most importantly, adding the proxy round does no harm: in the simplest settings where the clock auction alone performs well adding the proxy round should not distort the outcome. The proxy round simply expands the settings in which the auction performs well.

References