2-Player Signaling Games

- Informed and uninformed players
- Let the buyer beware
- Three varieties of market failure, and one variety of market success
Two-Player Signaling Games

A good item is worth $V$ to the buyer
A bad item is worth $W$ to the buyer

Any item, either good or bad, is offered at price $p$

$V > p > W$

The seller of a bad item must pay a cleanup cost, $c$

Caveat Emptor, extensive form

[Diagram showing the game tree with the following payoffs:

- If the item is good ($p$), the seller offers it for sale and gets $p, V - p$ with probability 0.5.
- If the item is bad ($p$), the seller offers it for sale and gets $p - c, W - p$ with probability 0.5.
- If the seller offers the item and the buyer accepts, they get the respective payoffs.
- If the buyer rejects, they get 0, 0.
- If the seller offers the item and the buyer rejects, they get 0, 0.
- If the buyer stops without an offer, they get 0, 0.
]
Caveat Emptor, playing the game

Price of the car: $4000

Value: good car (red card) $6000
       bad car (black card) $3000

Clean-up cost: $2000

Caveat Emptor, action sheet

Name:

<table>
<thead>
<tr>
<th>good/bad</th>
<th>action taken</th>
<th>payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Total Market Failure

All sellers, even the good-type sellers, fearing rejection by the buyers, withhold their goods from the market. The market ceases to function, even though gains from trade are available. An equilibrium where all informed players do the same thing is called a **pooling equilibrium**. Total market failure is an especially sinister pooling equilibrium.

Total Market Success

*Only* sellers with good items offer them for sale. Since all items offered for sale are good, buyers buy everything offered for sale and the market works perfectly. An equilibrium where the types of informed players do different thing is called a **separating equilibrium**. Here, the very act of offering the item for sale signals to the buyer that it is a good type.
Partial Market Success

All sellers offer their items for sale, good or bad. All buyers buy whatever is offered for sale. This is only a partial success: the market functions, but there are a lot of bad deals, which reduces market efficiency. This is another example of pooling equilibrium. Unlike total market failure, however, this pooling equilibrium does generate some gains from trade.

Near Market Failure

Some, but not all, bad-type sellers offer their items for sale. Buyers buy what is offered for sale with a certain probability, and reject what is offered with some probability. Thus, both buyers and bad-type sellers adopt a mixed strategy response to the imperfect information. In this market, total gains from trade are smaller than in complete or partial market success.
Sequential Equilibrium: Pure Strategies

- Sequential equilibrium as an extension of subgame perfect equilibrium
- Sequential equilibria that pool informed players
- Sequential equilibria that separate informed players

Partial market success, reaching player 2’s information set
Bayes’ Rule

Suppose, in a sequential equilibrium:
1 offers an item for sale no matter the item is good or bad.

By conditional probability,
\[ p(\text{good} \mid \text{offer}) = \frac{p(\text{good}) \cdot p(\text{offer} \mid \text{good})}{p(\text{offer})} \]

\[ p(\text{offer}) = p(\text{good}) \cdot p(\text{offer} \mid \text{good}) + p(\text{bad}) \cdot p(\text{offer} \mid \text{bad}) = 1 \]

Also, \( p(\text{offer} \mid \text{good}) = p(\text{offer} \mid \text{bad}) = 1 \)
\[ \therefore p(\text{good} \mid \text{offer}) = \frac{p(\text{good}) \times 1}{1} = p(\text{good}) \]

Similarly, \( p(\text{bad} \mid \text{offer}) = p(\text{bad}) \)

This is known as **Bayes’ Rule**

Partial market success, backward induction, player 1’s move

\[
\begin{array}{c}
\text{Offer for sale} \\
p, V - p \\
\text{Offer for sale} \\
p - c, W - p \\
\end{array}
\]

\[
\begin{array}{c}
0, 0 \\
\text{Stop} \\
0, 0 \\
\end{array}
\]

\[
\begin{array}{c}
1 \\
p(\text{good}) \\
\end{array}
\]

\[
\begin{array}{c}
1 \\
p(\text{bad}) \\
\end{array}
\]

\[
\begin{array}{c}
0 \\
\end{array}
\]
**Sequential equilibrium when** \( p > c \)

The following is a sequential equilibrium:
1. offers the item for sale if it is good
2. offers the item for sale if it is bad
3. buys whatever that is offered for sale

\[ p(\text{node g} \mid \text{offer}) = p(\text{good}) \text{ and } p(\text{node b} \mid \text{offer}) = p(\text{bad}) \]

\[ p(\text{node g} \mid \text{offer}) = \frac{p(\text{good} \mid \text{offer})}{p(\text{good} \mid \text{offer}) + p(\text{bad} \mid \text{offer})} = \frac{p(\text{good})}{p(\text{good}) + p(\text{bad})} = p(\text{good}) \]

Similarly, \( p(\text{node b} \mid \text{offer}) = p(\text{bad}) \)

**Sequential equilibrium when** \( p > c \)

\[ Eu_2 = p(\text{node g} \mid \text{offer}) (V - p) + p(\text{node b} \mid \text{offer}) (W - p) \]

\[ = p(\text{good} \mid \text{offer}) (V - p) + p(\text{bad} \mid \text{offer}) (W - p) \]

\[ = p(\text{good}) (V - p) + p(\text{bad}) (W - p) \]

Require
\[ p(\text{good}) (V - p) + p(\text{bad}) (W - p) > 0 \]

\[ \therefore p(\text{good}) \text{ must be sufficiently large.} \]

This is partial market success
Complete market success, reaching player 2’s information set

Complete market success, backward induction, player 1’s move
Sequential equilibrium when $c > p$

The following is a sequential equilibrium:
1 offers the item for sale if it is good
1 stops if the item is bad
2 buys whatever that is offered for sale

\[ p(\text{node g} | \text{offer}) = 1 \text{ and } p(\text{node b} | \text{offer}) = 0 \]

\[ p(\text{node g} | \text{offer}) = p(\text{good} | \text{offer}) \]
\[ = p(\text{good}) \times p(\text{offer} | \text{good}) / p(\text{offer}) = p(\text{good}) / p(\text{offer}) \]

\[ p(\text{offer}) = p(\text{good}) \times p(\text{offer} | \text{good}) + p(\text{bad}) \times p(\text{offer} | \text{bad}) \]
\[ = p(\text{good}) \Rightarrow p(\text{node g} | \text{offer}) = p(\text{good}) / p(\text{good}) = 1 \]

Similarly, \( p(\text{node b} | \text{offer}) = p(\text{bad} | \text{offer}) = 0 \)

Sequential equilibrium when $c > p$

\[ Eu_2 = p(\text{node g} | \text{offer}) (V - p) + p(\text{node b} | \text{offer}) (W - p) \]
\[ = p(\text{good} | \text{offer}) (V - p) + p(\text{bad} | \text{offer}) (W - p) \]
\[ = p(\text{good}) (V - p) + p(\text{bad}) (W - p) \]
\[ = 1 \times (V - p) + 0 \times (W - p) > 0 \]

This is complete market success
Complete market failure, backward induction, player 1’s move

The following sequential equilibrium is not based on information about the items and leads to complete market failure:

1 stops whatever the item is
2 says no to any item offered for sale

\[ p(\text{node g} \mid \text{offer}) = 0 \quad \text{and} \quad p(\text{node b} \mid \text{offer}) = 1 \]

\[ Eu_2 = p(\text{node g} \mid \text{offer}) \cdot (V - p) + p(\text{node b} \mid \text{offer}) \cdot (W - p) \]

\[ = 0 \times (V - p) + 1 \times (W - p) < 0 \]

This is complete market failure
Sequential Equilibrium: Mixed Strategies

- Sequential equilibria for markets on the verge of total failure
- A sequential equilibrium that partially separates and partially pools informed players
- A regime diagram for sequential equilibria

Caveat Emptor, near market failure
Sequential equilibrium for near market failure

In this case, \( p > c \) and
\[
Eu_2 = p(\text{good}) (V - p) + p(\text{bad}) (W - p) < 0
\]
Suppose, \( p(\text{good}) = p(\text{bad}) = .5 \),
\( V = $3000, p = $2000, W = 0 \) and \( c = $1000 \)
\[
dot \ p(\text{good}) (V - p) + p(\text{bad}) (W - p)
\]
\[
= .5 \times 1000 + .5 \times -2000 < 0
\]
The following is a sequential equilibrium:
1 offers for sale if the item is good
1 offers for sale with probability .5 if the item is bad
2 buys any item with probability .5
\( p(\text{node g | offer}) = 2/3 \) and \( p(\text{node b | offer}) = 1/3 \)

Verifying the sequential equilibrium for near market failure

\( p(\text{node g | offer}) = .5 / (.5 + .25) = 2/3 \)
\( p(\text{node b | offer}) = .25 / (.5 + .25) = 1/3 \)
\( Eu_2(\text{buy | offer}) \)
\[
= p(\text{node g | offer}) 1000 + p(\text{node b | offer}) (-2000)
\]
\[
= 2/3 \times 1000 + 1/3 \times (-2000) = 0
\]
\( Eu_1(\text{offer | good}) = .5 \times 2000 + .5 \times 0 = 1000 > 0 \)
\( Eu_1(\text{offer | bad}) = .5 \times 1000 + .5 \times -1000 = 0 \)
Caveat Emptor, regime diagram

\[ \Pr(\text{good})(V - p) + \Pr(\text{bad})(W - p) \]

- Partial market success
- Complete market success
- Near market failure

\[ c = p \]

The market for Lemons

- When price signals quality, and when it fails to signal quality
- A market fails when only lemons are offered for sale
- Examples from health insurance and used cars
The Market for Lemons

In a Market for Lemons,

A good item is worth $V$ to the buyer
A bad item is worth $W$ to the buyer
An item is offered at either a high price $p$ or a low price $q$
Where, $V - q > V - p > W - q > 0 > W - p$
The seller of a bad item must pay a cleanup cost, $c$
Sequential Equilibrium in a market for Lemons when $c > p$

The following is a sequential equilibrium:
1 charges a high price with a good item
1 charges a low price with a bad item
2 buys any item offered for sale
$p(\text{node } g \mid \text{high price}) = 1$
$p(\text{node } b \mid \text{low price}) = 1$

In this case, thanks to the separating sequential equilibrium, Lemons has a complete market success solution.

---

Sequential Equilibrium in a market for Lemons when $c = 0$

In this case, a bad item can mimic a good item for free and price no longer indicates quality

A buyer’s expected value at either price is negative:
$Eu_2(\text{buy} \mid \text{high price}) = p(\text{good})(V - p) + p(\text{bad})(W - p) < 0$
$Eu_2(\text{buy} \mid \text{low price}) = p(\text{good})(V - q) + p(\text{bad})(W - q) < 0$

In the very worst case the breaks down completely:
1 charges a low price with a good item
2 does not buy at either price
$p(\text{node } g \mid \text{low price}) = p(\text{good})$
$p(\text{node } g \mid \text{high price}) = p(\text{good})$
The lemons principle

The bad drives everything out of the market

Costly Commitment as a Signaling Device

- The principle of costly commitment
- Money-back-guarantees as costly commitments to solve the lemons problem
- Costly commitments in all walks of life
Money-Back Guarantee

Sequential Equilibrium when the money-back guarantee is costly enough

The bad-type seller has to reimburse the unlucky buyer

$V - W$

If $p - W - V < 0$, the following sequential equilibrium leads to complete market success:

1 charges a high price with a good item
1 charges a low price with a bad item
2 says yes to a high price
2 says no to a low price

$p(\text{node } g \mid \text{high price}) = 1$
$p(\text{node } g \mid \text{low price}) = 0$
Screening Games

- Uninformed player moves first
- Offer of contracts by uninformed player
- Subgame perfection
- Menu of contracts to achieve full market success
- Existence of separating equilibrium

Screening game

<table>
<thead>
<tr>
<th>Offer Contract</th>
<th>p(good)</th>
<th>p(bad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offer Contract</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Do not Offer Contract</td>
<td>0, 0</td>
<td>-v, 1</td>
</tr>
</tbody>
</table>

Payoffs:
- 0, 0
- v, 1
- -v, 1

Decision nodes:
- 1
- 0
Backwards induction in the screening game

Both buyers accept insurance contract since $1>0$ for each.

Insurance company has expected value:

$$E(V_1) = p(good)v + p(bad)(-v) = [p(good) - p(bad)]v$$

Hence insurance company offers contract if expected value is positive, or

$$p(good) > p(bad)$$

and does not offer contract if the expected value is negative,

$$p(good) < p(bad)$$

Backwards induction in the screening game

Full market success needs more complexity. Insurance company has two types of contract:

- contract I: low premium, large deductible
- contract II: high premium, low deductible

Then there could a subgame perfect equilibrium:

- insurance offers both contract
- good risk player: accept contract I, reject contract II
- bad risk player: reject contract I, accept contract II
Barbarians at the Gate

- The power of inside information in corporate takeovers
- Buying a corporation and buying a used car
- Bidding behavior that reveals an underlying signal
- The potential lemons problem in corporate takeovers

Repeated Signaling and Track Records

- Noisy signals and unknowable player types
- Bayesian updating during the probationary period
- Detailed calculations for one-period probation
- The trade-off between longer probation and the opportunity cost of ordinary performance
One-Period Probation

\[ p(\text{star}) = p(\text{ordinary}) = 0.5 \text{ and discount factor } = 0.95 \]

When the prospective partner is a star performer,
\[ EV = \Sigma(0.95)^t + [0.9 \times 1 + 0.1 \times 0] = 0.9 / (1 - 0.95) = 18 \]

When the prospective partner is a ordinary performer,
\[ EV = \Sigma(0.95)^t + [0.5 \times 1 + 0.5 \times 0] = 0.5 / (1 - 0.95) = 10 \]

When you let the prospective partner go,
\[ EV = [0.5 \times (18 - 1) + 0.5 \times (10 - 1)] = 13 \]
Deciding on performance in one trial

Strategy 1: Yes no matter what

\[ EV = (.5 \times 18 + .5 \times 10) = 14 \]

Strategy 2: Yes if a win, No if a loss

\[
p(\text{star} \mid \text{win}) = \frac{p(\text{star}) \cdot p(\text{win} \mid \text{star})}{p(\text{win})} = \frac{.9 \times .5}{.7} = \frac{9}{14}
\]

\[
p(\text{ordinary} \mid \text{win}) = 1 - p(\text{star} \mid \text{win}) = \frac{5}{14}
\]

\[
p(\text{star} \mid \text{loss}) = \frac{p(\text{star}) \cdot p(\text{loss} \mid \text{star})}{p(\text{loss})} = \frac{.5 \times .1}{.3} = \frac{1}{6}
\]

\[
p(\text{ordinary} \mid \text{loss}) = 1 - p(\text{star} \mid \text{loss}) = \frac{5}{6}
\]

\[
\therefore EV = .7 \times [9/14 \times 18 + 5/14 \times 10] + .3 \times 13 = 14.5^{45}
\]

Deciding on performance in one trial

Strategy 3: Yes if a loss, No if a win

\[ EV = .7 \times 13 + .3 \times [1/6 \times 18 + 5/6 \times 10] = 12.5 \]

Strategy 4: No, no matter what

\[ EV = [.5 \times (18 - 1) + .5 \times (10 - 1)] = 13 \]
Ten-Period Probation: ordinary performer

Ten-Period Probation: star performer
Hit and Run: Track Records in Hollywood

- Sony purchase of Columbia movie studio
- Asymmetric information in hiring directors Jon Peters and Peter Gruber
- Use of past performance as signal