Chapter 8

Repeated Games

Strategies and payoffs for games played twice

- Finitely repeated games
- Discounted utility and normalized utility
- Complete plans of play for 2×2 games played twice
- Trigger strategies
A 2×2 game played twice

First play of one-shot game

Second play

Strategies for playing a 2×2 game twice

<table>
<thead>
<tr>
<th>Strategy No.</th>
<th>Round 1</th>
<th>After (L, L)</th>
<th>After (L, R)</th>
<th>After (R, L)</th>
<th>After (R, R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>2</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>R</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>R</td>
<td>L</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>17</td>
<td>R</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>18</td>
<td>R</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>R</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>31</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>L</td>
</tr>
<tr>
<td>32</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
</tbody>
</table>
Zero-sum, 2-player games played more than once

- Repetition adds nothing new to a zero-sum game
- Zero-sum is no basis for a relationship

Variable sum games with a single equilibrium, played twice

- Selten’s theorem on unique subgame perfect equilibria
- Repetition by itself does not solve a credibility problem
- Finite repetition of Cournot market game
Prisoner’s Dilemma, played twice

```
Player 1  Do not Confess  Confess

Do not Confess
3, 3  0, 4

Confess
4, 0  2, 2
```

Prisoner’s Dilemma, backward induction (second play): Pay-off matrix

```
Player 2

Player 1  Do not Confess  Confess

Do not Confess
3, 3  0, 4

Confess
4, 0  2, 2
```
Prisoner’s Dilemma, backward induction (second play): Player 1’s strategy

Player 1
- Do not Confess
  - Player 2
    - Do not Confess: (3, 3)
    - Confess: (0, 4)
- Confess
  - Player 2
    - Do not Confess: (4, 0)
    - Confess: (2, 2)

Prisoner’s Dilemma, backward induction (second play): Player 2’s strategy

Player 2
- Do not Confess
  - Player 1
    - Do not Confess: (3, 3)
    - Confess: (0, 4)
- Confess
  - Player 1
    - Do not Confess: (4, 0)
    - Confess: (2, 2)
Prisoner’s Dilemma, second play, by backward induction

Player 2

Player 1

Do not Confess

Confess

Do not Confess

Confess

3, 3

0, 4

4, 0

2, 2

Prisoner’s Dilemma, first play: Get (2,2) in second play regardless of first

Confess

Confess

4, 0

2, 2

4, 0

2, 2

3, 3

0, 4

3, 3

0, 4

4, 0

2, 2

4, 0

2, 2

3, 3

0, 4

3, 3

0, 4

4, 0

2, 2

4, 0

2, 2
Stern’s Theorem

If a game with a unique equilibrium is played finitely many times, its solution is that equilibrium played each and every time.
OPEC Won’t Curb Oil Until Others Do

- OPEC’s quota system, 1973-1993
- The attempt to improve upon a one-shot Cournot equilibrium
- Finiteness of a resource and finiteness of a game

OPEC quotas

\[ P = \frac{1}{Q} \]

\[ P^+ \quad \text{OPEC target} \]

\[ P^* \quad \text{Market Outcome} \]

\[ Q \quad (\text{barrels/day}) \]

\[ \text{Total Quotas} \]

\[ \text{Demand} \]

\[ \text{Deviations} \]
Variable sum games with multiple equilibria, played finitely many times

- Multiple one-shot equilibria mean multiple subgame perfect equilibria
- Equilibria involving rotation
- Equilibria involving trigger strategies
- The Folk Theorem for finitely repeated 2-player games

Market Niches, the one-shot game

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>3, 3</td>
<td>B</td>
</tr>
<tr>
<td>A</td>
<td>1, 4</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>4, 1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>
Market Niches, the one-shot game:
Player 1’s strategy in pure strategy

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 3</td>
<td>1, 4</td>
<td>4, 1</td>
</tr>
<tr>
<td>0, 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Market Niches, the one-shot game:
Player 2’s strategy in pure strategy

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 3</td>
<td>1, 4</td>
<td>4, 1</td>
</tr>
<tr>
<td>0, 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Market Niches, the one-shot game: Two pure strategy equilibria

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3, 3</td>
<td>1, 4</td>
</tr>
<tr>
<td>B</td>
<td>4, 1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

Market Niches, played two or three times, average utility possibilities

\[
\begin{align*}
&u_1(1, 4) \\
&(1.5, 3) \\
&(3, 3) \\
&(2.5, 2.5) \\
&(2, 2) \\
&(3, 1.5) \\
&(4, 1) \\
\end{align*}
\]
Folk Theorem for Finitely Repeated Two-person Games

Suppose that a finitely repeated game has a one-shot equilibrium payoff vector that payoff dominates \( w \). Then all individually rational and feasible payoffs are supported in the limit as average payoffs of subgame perfect equilibria.

Folk Theorem: Market Niches played finitely many times
Infinitely repeated games: strategies and payoffs

- The “as if” interpretation of infinite repetition
- Complete plans for infinite play
- Discounting infinite series of payoffs
- Normalizing discounted discounted payoffs

Evaluating payoffs for an infinitely repeated game

Total payoff for player 1, \( u_1 = \sum R^t u_1(t) \)
\( t \) goes from 0 to \( \infty \); \( R \) is the discount factor where \( R < 1 \)

When \( u_1(t) = 1 \) for all \( t \), \( u_1 = 1 + R + R^2 + R^3 + \ldots \)

For \( 0 < R < 1 \), the series sums to
\( u_1 = 1/(1 - R) = 1 + R + R^2 + R^3 + \ldots \)

When \( u_1(t) = k \) for all \( t \), \( u_1 = k/(1 - R) \)

We can multiply the infinite sum of utilities by \( (1 - R) \) to keep this payoff on the same-scale as those of the one-shot game

Therefore, in general, \( u_1 = (1 - R) \sum R^t u_1(t) \)
Folk Theorem for Infinitely Repeated Games

Suppose that an infinitely repeated game has a payoff vector that exceeds $w_i$ for each player $i$. Then all individually rational and feasible payoffs that payoff dominate $w$ are supported as payoffs of subgame perfect equilibria, as long as discount factors are sufficiently close to 1. In particular, efficient payoff vectors are supported as payoffs of subgame perfect equilibria when discount factors are sufficiently close to 1.

Robert Axelrod’s Tournament

- Experiment to compare strategies
- Tit-for-tat strategy
- World War I use of poisonous gas
Infinitely repeated market games

- A one-shot Cournot equilibrium, repeated infinitely often, is a subgame perfect equilibrium path
- Better paying equilibria than one-shot Cournot
- Monopoly-like equilibria when firms attach enough importance to the future
- A Folk Theorem for infinitely repeated games
- Infinitely repeated Bertrand market games

Two-firm market game, played in infinite number of periods

Each period the market demand both firm 1 and 2 face is:
\[ P = 130 - Q \quad \text{and} \quad \text{constant marginal cost, } c = $10 \]

The Cournot Equilibrium is \( x_1^* = x_2^* = 40 \) and each firm’s profit = $1600

Suppose both firms play Cournot strategy infinitely often:
- every play: ship 40 units after every possible history

The infinite sum of payoffs, normalized, to each firm is
\[ \frac{(1 - R) \times 1600}{1 - R} = $1600 \]
Two-firm market game, played in infinite number of periods

Now, suppose firm 1 decides to charge $51 for each unit and as a result they sell only 39 units each period.

In each period $t$, $u_1(t) = (51 - 10) \times 39 = $1599

If firm 1 were to do this forever, it would get, $u_1 = $1599

This $u_1$ is $1$ lower than at the Cournot equilibrium

∴ Reducing output does not increase profit

Similarly, raising output does not increase profit either

Folk theorem: Infinitely repeated Cournot market game

\[
\begin{align*}
\text{IR} &= (0, 0) \\
A_1 &= (2000, 1600) \\
A_2 &= (1600, 2000) \\
C &= W = (1600, 1600) \\
(0, 3600) \\
(3600, 0)
\end{align*}
\]
Payoff possibilities, infinitely repeated Bertrand market game

\[ u_1 + u_2 = 1600 \]

\[ IR = w = (0, 0) = B \]

Price leadership in the ready-to-eat cereals industry

- The serious problem of equilibrium selection
- Using price leadership as an equilibrium selection device
- Kellogg’s as a price leader
- High rates of return in ready-to-eat cereals
Price Leadership

Under price leadership, one firm, the price leader, takes charge of the industry pricing policy. Every time a change in prices is called for by a change in economic fundamentals, the price leader makes the change. The members of the industry depend on the price leader to make the correct price changes, so that industry profits are as high as they can be.