Chapter 6
Noncooperative Market Games in Normal Form

Market game:
one seller and one buyer

- 2 players, a buyer and a seller
- Buyer receives red card
  - Ace=11, King = Queen = Jack = 10, 9, ..., 2
  - Number represents willingness to pay
- Seller receives black card
  - Ace=11, King = Queen = Jack = 10, 9, ..., 2
  - Number represents willingness to sell
- Buyer looks at her red card, and records a bid (2, 2.5, ..., 10.5, 11)
- Seller looks at his black card, and records an ask (2, 2.5, ..., 10.5, 11)
- Buyer and seller are matched by a random process
- If ask is less than or equal to bid, then deal at price \( p = \frac{(\text{bid} + \text{ask})}{2} \)
  - Buyer payoff = willingness to pay - price
  - Seller payoff = price - willingness to sell
- If ask is greater than the bid, then no deal
  - Seller payoff = Buyer payoff = 0
Market game:
many sellers and buyers

- Entire class, half buyers and half sellers
- Buyers receives red card
  - Ace = 11, King = Queen = Jack = 10, 9, ..., 2
  - Number represents willingness to pay
- Seller receives black card
  - Ace = 11, King = Queen = Jack = 10, 9, ..., 2
  - Number represents willingness to sell
- Buyer looks at her red card, and records a bid (2, 2.5, ..., 10.5, 11)
- Seller looks at his black card, and records an ask (2, 2.5, ..., 10.5, 11)
- Clearing price is found
  - Buyers with bids greater than clearing price trade
  - Sellers with asks less than clearing price trade
- If trade, then
  - Buyer payoff = willingness to pay - price
  - Seller payoff = price - willingness to sell
- If don’t trade, then
  - Seller payoff = Buyer payoff = 0

A market with 1 buyer and 1 seller

- Single unit that can be exchanged
- Price-taking behavior
- Honest revelation of potentially valuable information
1 buyer, 1 seller:
Market fundamentals

\[ P \]
\[ b^* \]
\[ \text{demand} \]
\[ 1 \]
\[ Q \]
\[ \text{buyer demand} \]

\[ P \]
\[ a^* \]
\[ \text{supply} \]
\[ Q \]
\[ \text{seller supply} \]

1 buyer, 1 seller:
Classical market equilibrium

\[ P \]
\[ b^* \]
\[ \text{supply} \]
\[ P^* \text{ between } a^* \text{ and } b^* \]
\[ a^* \]
\[ \text{demand} \]
\[ 1 \]
\[ Q \]
\[ \text{buyer demand} \]
A market with 1 buyer and 1 seller: The Market Game

- The rules of the game determine the outcome
- Open outcry market
- Producer’s surplus from trade
- Consumer’s surplus from trade

1 buyer, 1 seller market game: Outcome matrix

<table>
<thead>
<tr>
<th>Seller</th>
<th>Buyer</th>
<th>b = 2</th>
<th>b = 3</th>
<th>b = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = 4</td>
<td>P = 3</td>
<td>P = 3.5</td>
<td>P = 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q = 0</td>
<td>Q = 0</td>
<td>Q = 1</td>
<td></td>
</tr>
<tr>
<td>a = 3</td>
<td>P = 2.5</td>
<td>P = 3</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Q = 0</td>
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<td></td>
</tr>
<tr>
<td>a = 2</td>
<td>P = 2</td>
<td>P = 1.5</td>
<td>P = 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q = 1</td>
<td>Q = 1</td>
<td>Q = 1</td>
<td></td>
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</table>
Nash versus Perfectly Competitive Equilibrium

- Multiple Nash Equilibria
  - good equilibria
  - bad equilibrium
- Iterated (weak) dominance solution
- Path dependence: different order of elimination leads to different equilibria

1 buyer, 1 seller market game: Market game payoff matrix

<table>
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<tr>
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<tbody>
<tr>
<td>a = 4</td>
<td>0, 0</td>
<td>0, 0</td>
<td>2, 0</td>
</tr>
<tr>
<td>a# = 2</td>
<td>0, 0</td>
<td>1, 1</td>
<td>1.5, 0.5</td>
</tr>
<tr>
<td>a = 3</td>
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1 buyer, 1 seller market game: Strategy for seller

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1 buyer, 1 seller market game: Nash Equilibrium

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1 buyer, 1 seller market game: Iterated dominance solution

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1 buyer, 1 seller market game: Alternative elimination of strategies

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Market Games with many Buyers and Sellers

- Marginal pairs determine outcomes
- The role of the auctioneer
- Price formation on the stock exchange
  - limit orders
  - market orders

Demand and supply: Market game with four players

\[ P\star \text{ between 2 and 4} \]
Demand and supply: Strategic bid-ask arrays

Quantity Competition between two firms

- Cournot competition with 3 strategies
- Cournot equilibrium lies between monopoly and perfect competition
Cournot competition for two firms

Market Price, \( P = 130 - Q \) when \( Q \leq 130 \)
\[ = 0 \] otherwise

Market Quantity, \( Q = x_1 + x_2 + \ldots + x_n = \sum x_i \)

Quantity vector, \( x = (x_1, x_2, \ldots, x_n) \)
here \( x_i \) represents firm i’s quantity delivered to the market

\[ \therefore \text{For a market with 2 firms,} \]
\[ Q = x_1 + x_2 \text{ and } x = (x_1, x_2) \]

Constant average variable cost = \( c \)

Cournot competition for two firms: A firm’s profits

Firm i’s profits:
\[ u_i(x) = \text{revenue - cost} \]
\[ = P x_i - c x_i \]
\[ = (P - c) x_i \]
\[ \therefore u_1(x) = (P - c) x_1 \quad \text{and} \]
\[ u_2(x) = (P - c) x_2 \]
Cournot competition for two firms: Profits for any given level of output

Let’s take $x_1 = x_2 = 30$ and $c = 10$

\[ Q = 30 + 30 = 60 \quad \text{and} \]

\[ P = 130 - 60 = 70 \]

\[ u_1(x) = (70 - 10) \times 30 = 1800 \]

\[ u_2(x) = (70 - 10) \times 30 = 1800 \]

Combination of profits will be different for different $x$s.

Cournot competition, two firms: Payoffs from different production plans

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
<th>$X_2 = 30$</th>
<th>$X_2 = 40$</th>
<th>$X_2 = 60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = 30$</td>
<td>1800, 1800</td>
<td>1500, 2000</td>
<td>900, 1800</td>
<td></td>
</tr>
<tr>
<td>$X_1 = 40$</td>
<td>2000, 1500</td>
<td>1600, 1600</td>
<td>800, 1200</td>
<td></td>
</tr>
<tr>
<td>$X_1 = 60$</td>
<td>1800, 900</td>
<td>1200, 800</td>
<td>0, 0</td>
<td></td>
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</tbody>
</table>
Cournot competition, two firms:
Strategy for firm 1

Firm 2

Firm 1

\[
\begin{array}{ccc}
X_2 = 30 & X_2 = 40 & X_2 = 60 \\
X_1 = 30 & 1800, 1800 & 1500, 2000 & \textbf{900}, 1800 \\
X_1 = 40 & \textbf{2000}, 1500 & \textbf{1600}, 1600 & 800, 1200 \\
X_1 = 60 & 1800, 900 & 1200, 800 & 0, 0 \\
\end{array}
\]

Cournot competition, two firms:
Strategy for firm 2

Firm 2

Firm 1

\[
\begin{array}{ccc}
X_2 = 30 & X_2 = 40 & X_2 = 60 \\
X_1 = 30 & 1800, 1800 & \textbf{1500}, 2000 & 900, 1800 \\
X_1 = 40 & \textbf{2000}, 1500 & \textbf{1600}, 1600 & 800, 1200 \\
X_1 = 60 & 1800, \textbf{900} & 1200, 800 & 0, 0 \\
\end{array}
\]
Cournot competition, two firms: The equilibrium

Cournot Equilibrium:

\[ x^* = (40, 40) \]
\[ Q^* = 40 + 40 = 80 \]
\[ P^* = 130 - 80 = 50 \]
\[ u_1(x^*) = (50 - 10) \times 40 = 1600 \]
\[ u_2(x^*) = 1600 \]
Cournot competition for two firms

*Cournot competition between two firms leads to an outcome between monopoly and perfect competition*

---

Cournot Competition, two firms, Deriving a Firm’s Best Response

Utility function of firm i: \( u_i(x) = (P - c)x_i \)

Consider firm 1

Firm 1 maximizes its profit by producing up to the point where marginal profit equals zero:

\[
0 = \frac{\partial u_1}{\partial x_1} = (P - c) + x_1\frac{\partial P}{\partial x_1}
\]

\[
\Rightarrow 0 = (120 - x_1 - x_2) + x_1(-1)
\]

\[
\Rightarrow 0 = 120 - 2x_1 - x_2
\]
Cournot Competition, two firms, Deriving a Firm’s Best Response

Setting $x_1 = x_2$,
\[ 0 = 120 - 3x_1 \]
\[ \therefore x_1^* = 40 = x_2^* \]

As before,

**Cournot Equilibrium:**
\[ x^* = (40, 40) \]

Perfect Competitive Equilibrium for two firms

- Market price equals marginal cost
- In this market, marginal cost = $c = $10
- $Q = 130 - P = 130 - 10 = 120$
- \( x^* = (60, 60) \)
- Profit for any firm = $(10 - 10) \times 60 = 0$
Cournot Competition, two firms, Deriving a Firm’s Best Response

Setting $x_1 = x_2$,

\[ 0 = 120 - 3x_1 \]

\[ \therefore x_1^* = 40 = x_2^* \]

As before,

**Cournot Equilibrium:**

\[ x^* = (40, 40) \]

---

Perfect Competitive Equilibrium for two firms

- Market price equals marginal cost
- In this market, marginal cost = $c = 10$
- $Q = 130 - P = 130 - 10 = 120$
- $x^* = (60, 60)$
- Profit for any firm = $(10 - 10) \times 60 = 0$
Monopoly Equilibrium for two firms

- Market profits are as large as possible
- A monopoly will maximize total market profit, \( u = u_1 + u_2 = (P - c) Q \)
  \[ \Rightarrow \quad u = (120 - Q) Q \]
- For maximizing profit, marginal utility of producing one more unit needs to be zero
  \[ \Rightarrow \quad 0 = \frac{\partial u}{\partial Q} = 120 - 2Q \]
  \[ \therefore Q^* = 60 \quad \text{and} \]
  total profits = \((120-60) \times 60 = \$3600\)

Cournot equilibrium in the market

- Monopoly is associated with the highest price, lowest quantity, and highest profit
- Perfect Competition is associated with the lowest price, highest quantity, and zero profit
- Cournot equilibrium lies in between on all three dimensions
The Cournot Limit Theorem

- The Cournot limit theorem: the higher the number of firms, the closer Cournot equilibrium gets to perfect competition
- The Cournot limit is good for the economy

Cournot market game:
Market outcomes compared

![Diagram showing market outcomes compared between Monopoly equilibrium, Cournot equilibrium, and Perfectly competitive equilibrium.](#)
Finding Cournot best responses

- Firm 1’s first-order condition is:
  \[2x_1 + x_2 = 120\]
- Solving for \(x_1\) as a function of \(x_2\) yields firm 1’s best-response function:
  \[x_1 = f_1(x_2) = 60 - \frac{x_2}{2}\]
- Similarly, firm 2’s best-response function:
  \[x_2 = f_2(x_1) = 60 - \frac{x_1}{2}\]

Cournot best responses, \(x^*\) = Cournot equilibrium

\[x_2 = f_2(x_1) = 60 - \frac{x_1}{2}\]

\[(40, 40) = x^*\]
Cournot variations, including many firms

- For any firm, profit $u_i(x) = (P - 10)x_i$
- Since all firms face the same costs and sell identical products, the game is symmetric. The profit maximization strategy for all the firms will be the same.
- We will focus on firm 1 to derive the Cournot equilibrium

For any firm, profit $u_i(x) = (P - 10)x_i$.

Since all firms face the same costs and sell identical products, the game is symmetric. The profit maximization strategy for all the firms will be the same.

We will focus on firm 1 to derive the Cournot equilibrium.

Firm 1 wants to maximize profit, $u_1(x) = (P - 10)x_1$.

Firm 1 maximizes profit when

$$0 = \frac{\partial u_1}{\partial x_1} = (P - 10) + x_1(\frac{\partial P}{\partial x_1})$$

$$= 120 - \sum x_i - x_1$$

By Symmetry, $\sum x_i = nx_1$

Therefore, $0 = 120 - (n+1)x_1$.

$\Rightarrow x_1^* = 120/(n+1)$
Cournot variations, including many firms

- Market Quantity,
  \[ Q^* = \Sigma x_1 = nx_1 = \frac{120n}{n+1} \]

- Market Price,
  \[ P^* = 130 - Q^* = 130 - \left[ \frac{120n}{n+1} \right] \]

- As \( n \to \infty \Rightarrow P^* = $10 \) and \( Q^* = 120 \)

- Cournot equilibrium becomes perfect competition equilibrium as \( n \) goes to infinity

Consumer and Producer Surplus: Monopoly outcome
Consumer and Producer Surplus:
Cournot outcome

Consumer and Producer Surplus:
Perfect competition outcome
Are Coffee Prices Going up?

- The coffee accord of 1993
- Cartels vs. Cournot equilibrium

Reasons behind Campala agreement

- Coffee demand is price inelastic, with a price elasticity of demand of about -0.5
- Price elasticity = (% change in quantity)/ (% change in price)

  For the coffee market,
  
  -0.5 = -20% / % change in price
  
  ⇒ % change in price = 40%

- This price increase of 40% would be a real boon for coffee producing countries.
Price Competition Between Two Firms

- Price competition is different from quantity competition
- Price competition leads to marginal cost pricing with as few as two firms
- General Motors 0% interest car financing
Price competition between two firms

Market Quantity, \( Q = 130 - P \)

Market, \( Q = x_1 + x_2 + \ldots + x_n = \sum x_i \)

Price vector, \( p = (p_1, p_2) \)
here \( p_1 \) and \( p_2 \) are firm 1’s and 2’s prices respectively

\( x_1(p) \) is the demand facing firm 1

Firm 1’s profit, \( u_1(p) = (p_1 - c) x_1(p) \)

Similarly, Firm 2’s profit, \( u_2(p) = (p_2 - c) x_2(p) \)

Demand functions for the two firms

The demand curve for firm 1:

\[
\begin{align*}
x_1(p) &= 130 - p_1 \\
&= (130 - p_1)/2 \\
&= 0 \\
\end{align*}
\]
when \( p_1 < p_2 \)
when \( p_1 = p_2 \)
when \( p_1 > p_2 \)

The demand curve for firm 2:

\[
\begin{align*}
x_2(p) &= 130 - p_2 \\
&= (130 - p_2)/2 \\
&= 0 \\
\end{align*}
\]
when \( p_2 < p_1 \)
when \( p_2 = p_1 \)
when \( p_2 > p_1 \)
Bertrand demand, firm 1

\[ x_1 = 0 \]

\[ x_1 = 65 - P_1/2 \]

\[ x_1 = 130 - P_1 \]

Bertrand market game, two firms:
All payoffs in $

\begin{array}{ccc}
\text{Firm 2} & p_2 = \$70 & p_2 = \$50 & p_2 = \$10 \\
\text{Firm 1} & p_1 = \$70 & 1800, 1800 & 0, 3200 & 0, 0 \\
& p_1 = \$50 & 3200, 0 & 1600, 1600 & 0, 0 \\
& p_1 = \$10 & 0, 0 & 0, 0 & 0, 0
\end{array}
Bertrand market game, two firms:
Strategy for firm 1

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>$p_2 = $70</th>
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<td>$p_1 = $50</td>
<td>3200, 0</td>
<td>1600, 1600</td>
<td>0, 0</td>
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<tr>
<td>$p_1 = $10</td>
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Bertrand market game, two firms:
Strategy for firm 2

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Bertrand market game, two firms: Two Bertrand equilibria

There are two Bertrand equilibria

One is $p_1^* = p_2^* = 50$ -- this is the same as the Cournot equilibrium

The second one is at the rock-bottom price $p_1^* = p_2^* = 10$ -- in this case, profit is 0

Applying the sufficient condition of undominated strategies, the Nash equilibrium $\mathbf{p} = (50, 50)$ is the strategy
Bertrand market game, two firms: Two Bertrand equilibria

There are two Bertrand equilibria

One is $p_1^* = p_2^* = 50$ -- this is the same as the Cournot equilibrium

The second one is at the rock-bottom price $p_1^* = p_2^* = 0$ -- in this case, profit is 0

Applying the sufficient condition of undominated strategies, the equilibrium $p = (50, 50)$ is the strategy

---

**Bertrand Variations**

- Bertrand equilibrium with a cost advantage
- Bertrand equilibrium with many firms
Bertrand Limit Theorem

When \( n \) is greater than or equal to 2, all products are perfect substitutes, and no firm has a cost advantage, then the Bertrand game equilibrium implies that price equals marginal cost.

Market Games with Differentiated Products

- Price and quantity competition when products are differentiated
- Cournot and Bertrand equilibrium still different, but the difference is muted
- Monopolistic competition as the limit of market game equilibrium
Differentiated Products

All differentiated products have one thing in common: if the price is slightly above the average price in the market, a firm doesn’t lose all its sales.

Two firms in a Bertrand competition

- The demand function faced by firm 1: \( x_1(p) = 180 - p_1 - (p_1 - \text{average price}) \)
- The demand function faced by firm 2: \( x_2(p) = 180 - p_2 - (p_2 - \text{average price}) \)
Two firms in a Bertrand competition

- Firm 1 has profits
  \[ u_1(p_1, p_2) = (p_1 - 20) x_1 \]
  \[ = (p_1 - 20) (180 - 2p_1 + \text{average price}) \]
  \[ = (p_1 - 20) (180 - 1.5p_1 + 0.5p_2) \]

- Firm 2’s profit function
  \[ u_2(p_1, p_2) \]
  \[ = (p_2 - 20) (180 - 1.5p_2 + 0.5p_1) \]

Maximizing profits

Firm 1 maximizes its profits when its marginal profit is zero:
\[ 0 = \frac{\partial u_1}{\partial p_1} \]
\[ = (p_1 - 20) (-1.5) + (180 - 1.5p_1 + 0.5p_2) \]
\[ \Rightarrow 0 = 210 - 3p_1 + 0.5p_2 \]
Firm 1’s best response function:
\[ p_1 = f_1(p_2) = 70 + \frac{p_2}{6} \]
Similarly, Firm 2’s best response function:
\[ p_2 = f_2(p_1) = 70 + \frac{p_1}{6} \]
Bertrand best responses, two firms, differentiated products

\[ p_1 = f_1(p_2) = 70 + \frac{p_2}{6} \]
\[ p_2 = f_2(p_1) = 70 + \frac{p_1}{6} \]

\[ p^* = (84, 84) \]

Bertrand Equilibrium

- The Bertrand equilibrium of the market game is located at (84, 84)
- The market price is $84, significantly higher than the marginal price which is given at $20
- Each firm sales (180 - 84) units = 96 units
- Each firms profit = (84 - 20) × 96
  = $6144
- Therefore, each firm could spend over $6000 in differentiating its products and can still come out ahead
Bertrand competition with n firms

- Firm 1’s market demand: \( x_1 = 180 - p_1 - \frac{n}{2} (p_1 - \text{average price}) \)
- Firm 1’s profit function: \( u_1(p) = (p_1 - 20)x_1 \)
- When the first order condition is satisfied:
  \[ 0 = \frac{\partial u_1}{\partial p_1} = (p_1 - 20)(-1 - \frac{n}{2} + \frac{1}{2}) + x_1 \]
  \[ \Rightarrow K(p_1 - 20) = 180 - p_1 \]
  \[ \Rightarrow p_1^* = \frac{180}{K+1} + \frac{20K}{K+1} \]

Bertrand competition with infinite number of firms

- As \( n \to \infty \), \( K \to \infty \) and \( \frac{1}{K} \to 0 \)
- Taking limit of \( p_1^* \) as \( n \) goes to infinity:
  \[ \lim p_1^* = \lim \frac{20}{1 + \frac{1}{K}} = 20 \]
- In this limit, price is equal to marginal cost and profits vanish.
- This limit is monopolistic competition.
Bertrand Competition among Differentiated Products in the Cigarette Industry

- Discount brands penetrate the market, 1981-now
- The price war between discount brands and name brands
- The Marlboro price cut of 1993

Appendix. Uniqueness of Equilibrium

- The contraction mapping theorem
- Best response mappings as contraction mappings
- Best response mapping fixed points are game equilibria