Chapter 5

n-Person Games in Normal Form

Fundamental Differences with 3 Players: the Spoilers

- Counterexamples
- The theorem for games like Chess does not generalize
- The solution theorem for 0-sum, 2-player games does not generalize
- A player playing the spoiler
Indeterminate three-person game

Top

3

Bottom

w, l, l

1, l, w, l

Multiple solutions, 3-person, zero-sum game

3-Top

1, -1/2, -1/2

3-Bottom

-1/2, 1, -1/2
Competitive Advantage and Market Niche with 3 Players

- The row-column matrix representation for 3 players
- Games where no player plays the spoiler
- Pure and mixed strategy equilibria for 3-player games

Competitive Advantage, three firms

Firm 3 - New Technology
Firm 3 - Stay Put

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Technology</td>
<td>New Technology</td>
</tr>
<tr>
<td>0, 0, 0</td>
<td>a/2, -a, a/2</td>
</tr>
<tr>
<td>-a, a/2, a/2</td>
<td>-a/2, -a/2, a</td>
</tr>
</tbody>
</table>

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<tr>
<td>New Technology</td>
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</tr>
<tr>
<td>a/2, a/2, -a</td>
<td>a, -a/2, -a/2</td>
</tr>
<tr>
<td>-a/2, -a, -a/2</td>
<td>0, 0, 0</td>
</tr>
</tbody>
</table>
Competitive Advantage, three firms:

Strategy for Firm 1

Firm 3 - New Technology
New Technology
0, 0, 0
\( \frac{a}{2}, -a, \frac{a}{2} \)
-a, a/2, a/2
-a/2, -a/2, a

Firm 3 - Stay Put
Stay Put

Firm 2
New Technology
\( \frac{a}{2}, a/2, -a \)
-a, -a/2, -a/2

Firm 1
New Technology
0, 0, 0
\( \frac{a}{2}, -a, \frac{a}{2} \)
-a, a/2, a/2

Stay Put
-a, a/2, a/2
-a/2, -a/2, a

Strategy for Firm 2

Firm 3 - New Technology
New Technology
0, 0, 0
\( \frac{a}{2}, -a, \frac{a}{2} \)
-a, a/2, a/2

Firm 3 - Stay Put
Stay Put

Firm 2
New Technology
\( \frac{a}{2}, a/2, -a \)
-a, -a/2, -a/2

Firm 1
New Technology
0, 0, 0
\( \frac{a}{2}, -a, \frac{a}{2} \)
-a, a/2, a/2

Stay Put
-a, a/2, a/2
-a/2, -a/2, a
Competitive Advantage, three firms:
Strategy for Firm 3

Firm 3 - New Technology

Firm 3 - Stay Put

Firm 2

Firm 1

New Technology

Stay Put

0, 0, 0

a/2, -a, a/2

-a, a/2, a/2

-a/2, -a/2, a

-a, a/2, 0

-a/2, -a/2, -a/2

Stay Put

-a, -a/2, -a

-a/2, a, -a/2

0, 0, 0

Competitive Advantage, three firms: The Nash equilibrium

Firm 3 - New Technology

Firm 3 - Stay Put

Firm 2

Firm 1

New Technology

Stay Put

0, 0, 0

a/2, -a, a/2

-a, a/2, a/2

-a/2, -a/2, a

-a, 0, 0

-a/2, -a, -a/2

Stay Put

-a, a, -a/2

-a/2, a, -a/2

0, 0, 0
Market Niche for three firms

Firm 3 - Enter

Firm 3 - Stay Out

Enter

Stay Out

Enter

Stay Out

Firm 2

Firm 1

Enter

-50, -50, -50

-50, 0, -50

0, -50, -50

0, 0, 100

0, -50, -50

0, 0, 100

100, 0, 0

0, 0, 0

Enter

100, 0, 0

0, 0, 0

Market Niche, three firms:
Strategy for Firm 1

Firm 3 - Enter

Firm 3 - Stay Out

Enter

Stay Out

Enter

Stay Out

Firm 2

Firm 1

Enter

-50, -50, -50

-50, 0, -50

0, -50, -50

0, 0, 100

0, -50, -50

0, 0, 100

0, 100, 0

0, 0, 0

100, 0, 0

0, 0, 0

0, 100, 0

0, 0, 0
Market Niche, three firms:
Strategy for Firm 2

Firm 3 - Enter

Firm 2

Firm 1

Enter

-50, -50, -50

-50, 0, -50

Stay Out

0, -50, -50

0, 0, 100

Firm 3 - Stay Out

Firm 2

Firm 1

Enter

-50, -50, 0

0, 100, 0

Stay Out

0, 0, 0

0, 0, 0

Market Niche, three firms:
Strategy for Firm 3

Firm 3 - Enter

Firm 2

Firm 1

Enter

-50, -50, -50

-50, 0, -50

Stay Out

0, -50, -50

0, 0, 100

Firm 3 - Stay Out

Firm 2

Firm 1

Enter

-50, -50, 0

100, 0, 0

Stay Out

0, 0, 0

0, 0, 0
Market Niche, three firms:
Three pure strategy equilibria

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>-50, -50, -50</td>
<td>-50, 0, -50</td>
</tr>
<tr>
<td>Stay Out</td>
<td>0, -50, -50</td>
<td>0, 0, 100</td>
</tr>
</tbody>
</table>

Mixed Strategy equilibrium in Market Niche with 3 players

From the standpoint of the market, the distribution of number of firms in the market niche, according to mixed strategy equilibria is as follows:

- $\rho(3$ firms enter$) = .08$
- $\rho(2$ firms enter$) = .31$
- $\rho(1$ firm enters$) = .42$
- $\rho(No$ firm enters$) = .19$
3-Player Versions of Coordination, Deal-Making, and Advertising

- Video System Coordination with 3 firms
- Let’s Make a Deal with 3 firms
- Cigarette Advertising on Television with 3 firms

Video System Coordination, three firms: The payoff matrices

<table>
<thead>
<tr>
<th></th>
<th>Firm 3 - Beta</th>
<th>Firm 3 - VHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm 1</td>
<td>Beta</td>
<td>VHS</td>
</tr>
<tr>
<td>Beta</td>
<td>1, 1, 1</td>
<td>0, 0, 0</td>
</tr>
<tr>
<td>VHS</td>
<td>0, 0, 0</td>
<td>0, 0, 0</td>
</tr>
</tbody>
</table>

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<tr>
<th></th>
<th>Firm 2</th>
<th>Firm 1</th>
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<tr>
<td>Firm 1</td>
<td>Beta</td>
<td>VHS</td>
</tr>
<tr>
<td>Beta</td>
<td>0, 0, 0</td>
<td>0, 0, 0</td>
</tr>
<tr>
<td>VHS</td>
<td>0, 0, 0</td>
<td>1, 1, 1</td>
</tr>
</tbody>
</table>
Video System Coordination, three firms: Strategy for Firm 1

Firm 3 - Beta

Firm 3 - VHS

Firm 2

Firm 1

Beta

VHS

1, 1, 1

0, 0, 0

0, 0, 0

0, 0, 0

1, 1, 1

0, 0, 0

0, 0, 0

0, 0, 0

Video System Coordination, three firms: Strategy for Firm 2

Firm 3 - Beta

Firm 3 - VHS

Firm 2

Firm 1

Beta

VHS

1, 1, 1

0, 0, 0

0, 0, 0

0, 0, 0

1, 1, 1

0, 0, 0

0, 0, 0

0, 0, 0

Video System Coordination, three firms: Strategy for Firm 3

Video System Coordination, three firms: Two pure strategy equilibria
Let’s make a deal, three players:
Payoffs in millions of dollars

Player 3 - Yes

Player 3 - No

Let’s make a deal, three players:
Strategy for player 1
Let’s make a deal, three players:
Strategy for player 2

Let’s make a deal, three players:
Strategy for player 3
Let’s make a deal, three players: Five pure strategy equilibria

Player 3 - Yes

Player 2

Yes

No

Player 1

Yes

No

Yes

No

5, 5, 5

0, 0, 0

0, 0, 0

0, 0, 0

0, 0, 0

Stonewalling Watergate

- Watergate as a 3-person Prisoner’s Dilemma
- Strictly dominant strategies and uniqueness of equilibrium
- Equilibria which are bad for the players
Stonewalling Watergate: D = Dean, E = Ehrlichman, H = Halderman

**Stonewall Talk**

<table>
<thead>
<tr>
<th></th>
<th>E - Stonewall</th>
<th>H - Talk</th>
</tr>
</thead>
<tbody>
<tr>
<td>D - Stonewall</td>
<td>-3, -3, -3</td>
<td>-5, -2, -2</td>
</tr>
<tr>
<td>D - Talk</td>
<td>-2, -5, -5</td>
<td>-2, -2, -5</td>
</tr>
</tbody>
</table>

**Stonewalling Watergate: Strategy for Dean**

<table>
<thead>
<tr>
<th></th>
<th>E - Stonewall</th>
<th>H - Talk</th>
</tr>
</thead>
<tbody>
<tr>
<td>D - Stonewall</td>
<td>-3, -3, -3</td>
<td>-5, -2, -5</td>
</tr>
<tr>
<td>D - Talk</td>
<td>-2, -5, -5</td>
<td>-2, -2, -5</td>
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</tbody>
</table>

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Stonewalling Watergate: Strategy for Ehrlichman

Stonewalling Watergate: Strategy for Halderman
Stonewalling Watergate: The Nash equilibrium

H - Stonewall
E
D
H - Talk

Stonewall
-3, -3, -3
-5, 2, -5
-2, -5, -5
-2, 2, -5

Talk

Stonewall
-5, -5, -2
-5, -2, -2

E
D
Hang Talk

Symmetry and Games with Many Players

- A compact notation for utility functions
- A generalized symmetry sufficient condition
- A symmetric game may have asymmetric equilibria
Solving Symmetric Games with Many Strategies

- A test for when a game is symmetric
- Symmetry makes games easier to solve
- Solving a game of common interest by exploiting the symmetry of the game
- The Nash Demand Game

The Nash demand game: the payoff matrix
The Nash demand game: player 1’s strategy

The Nash demand game: player 2’s strategy
The Nash demand game: Nash equilibrium

Stag Hunt

- Game requiring cooperation for efficient outcome
- Adding third player leads to qualitatively different outcome
  - additional Nash equilibria
  - asymmetric outcomes
  - possibility of free riding
Stag Hunt, two hunters: 
The payoff matrix

<table>
<thead>
<tr>
<th></th>
<th>hunt big</th>
<th>hunt small</th>
</tr>
</thead>
<tbody>
<tr>
<td>hunt big</td>
<td>3, 3</td>
<td>0, 1</td>
</tr>
<tr>
<td>hunt small</td>
<td>1, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

Stag Hunt, two hunters: 
Strategy for hunter 1

<table>
<thead>
<tr>
<th></th>
<th>hunt big</th>
<th>hunt small</th>
</tr>
</thead>
<tbody>
<tr>
<td>hunt big</td>
<td>3, 3</td>
<td>0, 1</td>
</tr>
<tr>
<td>hunt small</td>
<td>1, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>
Stag Hunt, two hunters:
Strategy for hunter 2

<table>
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<tr>
<th></th>
<th>Hunt big</th>
<th>Hunt small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunt big</td>
<td>(3, 3)</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>Hunt small</td>
<td>(1, 0)</td>
<td>(1, 1)</td>
</tr>
</tbody>
</table>

Stag Hunt, two hunters:
The equilibrium

<table>
<thead>
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<th></th>
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<th>Hunt small</th>
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<tbody>
<tr>
<td>Hunt big</td>
<td>(3, 3)</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>Hunt small</td>
<td>(1, 0)</td>
<td>(1, 1)</td>
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</table>
Stag Hunt, three hunters:
The payoff matrix

<table>
<thead>
<tr>
<th></th>
<th>hunter 2: hunt big</th>
<th>hunter 2: hunt small</th>
</tr>
</thead>
<tbody>
<tr>
<td>hunter 1: hunt big</td>
<td>3, 3, 3</td>
<td>3, 5, 3</td>
</tr>
<tr>
<td></td>
<td>5, 3, 3</td>
<td>1, 1, 0</td>
</tr>
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</table>

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<tr>
<th></th>
<th>hunter 2: hunt big</th>
<th>hunter 2: hunt small</th>
</tr>
</thead>
<tbody>
<tr>
<td>hunter 1: hunt big</td>
<td>3, 3, 5</td>
<td>0, 1, 1</td>
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<td></td>
<td>1, 0, 1</td>
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Stag Hunt, three hunters:
Strategy for hunter 1

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<th>hunter 2: hunt small</th>
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<tbody>
<tr>
<td>hunter 1: hunt big</td>
<td>3, 3, 3</td>
<td>3, 5, 3</td>
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<tr>
<td></td>
<td>5, 3, 3</td>
<td>1, 1, 0</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>hunter 2: hunt big</th>
<th>hunter 2: hunt small</th>
</tr>
</thead>
<tbody>
<tr>
<td>hunter 1: hunt big</td>
<td>3, 3, 5</td>
<td>0, 1, 1</td>
</tr>
<tr>
<td></td>
<td>1, 0, 1</td>
<td>1, 1, 1</td>
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</tbody>
</table>
Stag Hunt, three hunters:
Strategy for hunter 2

<table>
<thead>
<tr>
<th>Hunter 1</th>
<th>Hunter 2</th>
<th>Hunter 3: Hunt Big</th>
<th>Hunter 3: Hunt Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunt Big</td>
<td>3, 3, 3</td>
<td>3, 5, 3</td>
<td>3, 3, 5</td>
</tr>
<tr>
<td>Hunt Small</td>
<td>5, 3, 3</td>
<td>1, 1, 0</td>
<td>1, 0, 1</td>
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Stag Hunt, three hunters:
Strategy for hunter 3

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<tr>
<th>Hunter 1</th>
<th>Hunter 2</th>
<th>Hunter 3: Hunt Big</th>
<th>Hunter 3: Hunt Small</th>
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<tbody>
<tr>
<td>Hunt Big</td>
<td>3, 3, 3</td>
<td>3, 5, 3</td>
<td>3, 3, 5</td>
</tr>
<tr>
<td>Hunt Small</td>
<td>5, 3, 3</td>
<td>1, 1, 0</td>
<td>1, 0, 1</td>
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Stag Hunt, three hunters: Nash equilibria

The Tragedy of the Commons

- Games played on a commons
- The equilibrium of such a game has a tragic outcome
- Externalities
- First Welfare Theorem
- The case of the Geysers of Northern California
Tragedy of the Commons: score sheet

Payoff = 5(10 - x_i) + x_i (23 - 0.25 \Sigma x_i)

<table>
<thead>
<tr>
<th>strategy (x_i)</th>
<th>payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Tragedy of the Commons: Commons production function
Tragedy of the Commons

F(X) vs. X

Nash, 5 players
Nash, 10 players
Ultimate Tragic NE
F(X)/X = 0.1

Economic Zone
Uneconomic Zone

Tragedy of the Commons

F(X)

average product
efficient outcome
tragic outcome
marginal product

0.1
0
6
12

53
54
Appendix. Tragedy of the Commons in the Laboratory

- Playing a game in a behavior laboratory
- Tragic outcomes of a game played on a commons in a laboratory
- Unexplained phenomena