Chapter 1

An Introduction to Games and Their Theory

What is a Game?

- Varieties of games
  - board, card, video, field
  - economic games: bargaining, auctions

- Features of a game
  - rule-governed
  - strategy matters
  - outcomes
  - strategic interdependence
What is Game Theory, and Why?

- Game theory is the science of strategic interdependence
- Game theory is like the calculus
- Examples of game theory at work: market design
  - FCC spectrum auctions
  - electricity restructuring
- Personal uses of game theory
  - better strategies
  - better management
  - better understanding

1-player Games with Perfect Information

- Perfect Information
- Extensive form of a game (tree diagram)
- Features of the extensive form
  - endpoints
  - nodes
  - information sets
  - branches
  - payoffs
- Solving a game by backward induction
- A strategy is a complete plan of action
- Normal form of a game (matrix diagram)
The Maze

Entrance

Pot of Gold, Extensive Form

0

Left

Right

Up

Down

M

b

a

1

a

b

1

0
Pot of Gold, Backward Induction

Pot of Gold, normal form

Player 1

<table>
<thead>
<tr>
<th>Action at a, action at b</th>
<th>0</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up at a, left at b</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Up at a, right at b</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Down at a, left at b</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Down at a, right at b</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
A 2-Player Game with Perfect Information

- 2-players solving a Pot of Gold by backward induction
- 2-player Pot of Gold in normal form

Two-person Pot of Gold, extensive form

```
Player 1
\[ \begin{array}{c}
\text{Down} \\
\text{Up} \\
\text{Right} \\
\text{Left} \\
\end{array} \]
```

Player 2

```
\[ \begin{array}{c}
\text{Left} \\
\text{Right} \\
(0, 0) \\
(M/2, M/2) \\
\end{array} \]
```
Two-person Pot of Gold, normal form

Games like Chess

- 4 features of a game like Chess
  - 2-players
  - finite
  - perfect information
  - win-lose-or-draw
- the theorem for such games and its proof for a 2X2 games
- Strategy is all that matters in games like Chess
- Good strategies and bad strategies
Games like Chess

(Tic-Tac-Toe)

• Two players, 3×3 board

• Player 1 puts an ‘X’ in one of the empty cells of the game
• Player 2 puts an ‘O’ in one of the empty cells

• Both players alternate putting ‘X’s and ‘O’s into empty cells

• The first player to fill a row, column or diagonal wins
Games like Chess: Tic-Tac-Toe

Dominance of Strategies

- Concepts to compare strategies
  - weak dominance
  - strict dominance
- Players can use dominance arguments to choose strategies
- Dominance to solve games
  - weakly dominance solvable games
  - strict dominance solvable games
Strictly dominance solvable game

Extensive Form, Normal Form, and Coalition Function Form

- The three forms of a game
  - extensive form
  - normal form (strategic form)
  - coalition function
- Different games in extensive form may have the same normal form
- Coalition function form is used to study the gains of cooperation
Games nearly like Chess

(a) Left
(d, d)

(l, w)

(w, l)

(b) Left
(d, d)

(l, w)

(w, l)

Games in normal form

Player 1

Left

(d, d)

(d, d)

Right

(l, w)

(w, l)

Player 2
Two-person Pot of Gold, coalition function form

\[ u_1(0,0) \quad (0,M) \quad (M,0) \]