# Activity Rules for the Combinatorial Clock Auction

Lawrence M. Ausubel and Peter Cramton*

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***Preliminary***

*Abstract*

In the original proposal for the combinatorial clock auction (Ausubel, Cramton and Milgrom, 2006), a revealed-preference approach was taken to limiting bidders' activity, based on their earlier activity. However, empirical implementations of the CCA to spectrum auctions have tended to place most or all reliance on a monotonicity condition in eligibility points. This paper proposes activity rules which strike a balance between revealed preference and eligibility-point monotonicity. For the clock auction stage, we propose a hybrid revealed-preference/eligibility-point approach, in which the current round's bid should satisfy a simplified revealed-preference constraint relative to prior rounds' bids, but with an eligibility-point safe harbor. For the supplementary round, we propose a tightening of Ofcom's relative cap, in which supplementary bids must satisfy revealed preference relative to the final clock bid, as well as relative to bids in all eligibility-reducing rounds in which the bidder's eligibility went below the eligibility points associated with the bid in question.

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Send comments to:

Lawrence M. Ausubel
Department of Economics
University of Maryland
Tydings Hall, Room 3105
College Park, MD 20742–7211

ausubel@econ.umd.edu
+1 301 405 3495

Peter Cramton
Department of Economics
University of Maryland
Tydings Hall, Room 3105
College Park, MD 20742-7211

pcramton@gmail.com
+1 240 479 9345

* Larry Ausubel and Peter Cramton are Professors of Economics at the University of Maryland.

# 1. Introduction

The combinatorial clock auction (CCA), also known as the package clock auction or the clock-proxy auction, comprises a dynamic clock auction stage followed by a final, sealed-bid combinatorial round. The CCA, which is rapidly supplanting the simultaneous multiple round auction (SMRA) as the standard for spectrum auctions, was originally proposed in an academic paper by Ausubel, Cramton and Milgrom (2006). Other closely-related theoretical work includes papers by Ausubel and Milgrom (2002), Parkes and Ungar (2000), Parkes (2001), Porter, Rassenti, Roopnarine and Smith (2003), Ausubel (2004, 2006), Hoffman, Menon, van den Heever and Wilson (2006), Day and Raghavan (2007), Day and Milgrom (2008), Day and Cramton (2012), and Harsha, Barnhart, Parkes, and Zhang (2010). The pioneering work in introducing the CCA empirically was done by Ofcom (the UK communications regulator), which conducted the first two full CCAs in February and May 2008. A description of the design decisions in implementing the CCA and the development of theoretical results is in Cramton (2009).

In the original proposal for the CCA, a revealed-preference approach was taken to limiting bidders' activity, based on their earlier activity. However, owing to the potential complexity of revealed preference for bidders, the empirical implementations of the CCA to spectrum auctions have tended to place most or all reliance on a monotonicity condition in eligibility points.

In its March 2011 consultation document, Ofcom proposed activity rules based on eligibility points in the primary rounds (the clock stage), and then a relative cap on supplementary bids (Annexes 7-13, p. 69). While the relative cap is an important step in the direction of implementing revealed-preference considerations, there are at least two issues with the Ofcom (2011) approach:

1. The activity rule for the clock stage (Eligibility-Point Monotonicity) prevents a bidder from placing bids on her most preferred package whenever the most preferred package exceeds her eligibility. For example, the bidder may reduce her eligibility early in the auction but then need to expand her eligibility when the price in a category she is bidding on increases much more than the price of a substitute category requiring more eligibility points. This prevents the bidder from expressing her true preferences until the supplementary round.
2. The activity rule for the supplementary round (Relative Cap) fails to satisfy a desirable property that guarantees that the tentative allocation of the final clock round is unchanged as a result of the supplementary round when there are no unallocated items in the final clock round. Also, it is difficult for bidders to determine how to bid to guarantee winning the final clock package when items are unallocated in the final clock round.

In this short paper, we propose to resolve both of these problems by proposing hybrid activity rules which strike a balance between revealed preference and eligibility-point monotonicity.

## 2. Informal Description of the Proposed Activity Rules

*Informal description of the clock stage activity rule* ("Simplified RP/Eligibility-Point Hybrid"): In any clock round, the bidder can bid on a larger package than would be permitted by the bidder's current eligibility, provided that the package satisfies "revealed preference" with respect to each prior round's bid in which eligibility was reduced. (However, bidding on a larger package does not increase the bidder's eligibility in subsequent rounds.) At the same time, the bidder can always place a bid for any package that is within the bidder's current eligibility.

There are two ways for a bidder to think about this rule.

Consider a bidder who likes the simplicity of an eligibility-point monotonicity rule. Under our proposal, such a bidder can continue to bid just as she would bid under eligibility-point monotonicity. The only difference is that the bidder is given some extra flexibility to bid on a larger package, provided that the larger package satisfies revealed preference; i.e. the bid involves a switch to a package that has become relatively less expensive. Note that, under a pure revealed preference rule, a bidder may find it difficult to figure out how to correct a bid submission that violates a revealed preference constraint. However, under the Simplified RP/Eligibility-Point Hybrid rule, if the bidder is prevented from bidding on a larger package due to a violation of revealed preference, the bidder knows at least one straightforward way to correct the violation: she can reduce the size of the package until it satisfies eligibility-point monotonicity.

Consider a bidder who has a consistent model of her values for all packages and adopts the strategy of always bidding on her most preferred (i.e. most profitable) package in every round of the clock stage. Such a bidder will never be constrained by the activity rule. Moreover, if for some reason the bidder's values change during the auction, then it is possible that a revealed preference constraint will now bind, but the bidder knows that she can always fall back to a package whose size in eligibility points is consistent with her eligibility.

*Informal description of the supplementary round activity rule* ("Simplified Revealed-Preference Cap"): All supplementary bids must satisfy revealed preference with respect to the bidder's final clock package. In addition, supplementary bids for any packages that are larger than the final clock package must satisfy revealed preference with respect to each clock round that resulted in a reduction of eligibility, beginning with the last round in which the bidder had sufficient eligibility to bid on a given package.

Effectively, our proposed activity rule for the supplementary round strengthens the Relative Cap by applying a revealed-preference constraint relative to additional rounds. Under the Relative Cap, supplementary bids for packages that are larger than the final clock package must satisfy revealed preference with respect to the last round in which the bidder had sufficient eligibility to bid on the package. Under the Simplified Revealed-Preference Cap, supplementary bids for packages that are larger than the final clock package must also satisfy revealed preference with respect to each eligibility-reducing clock round after the last round in which the bidder had sufficient eligibility to bid on the package, as well as with respect to the final clock round.

The rule proposed for clock rounds enables the bidder to bid on her most preferred package throughout the clock rounds, thereby improving price discovery (more revelation of relevant marginal value information) and making the final clock allocation a better predictor of the auction outcome. Further, as we will see, the rule proposed for the supplementary round guarantees that the final clock allocation will not change if there are no unallocated items—each winner is guaranteed to win her final clock package without making any supplementary bids. And if there are unallocated items, then each winner can guarantee winning at least her final clock package with a supplementary bid that exceeds the dollar amount of her final clock package by the final clock price of the unallocated items. Finally, the rule constrains supplementary bids that have no chance of winning but would increase the payments of rivals.

The emphasis on revealed preference with respect to the final clock package motivates the bidder to bid on her most preferred package in the final clock round to improve her chances of winning her most preferred package. Since the bidder does not know which round will be the final clock round, there is a persistent motivation to bid on the most preferred package throughout the clock stage. This behavior is exactly what reveals the bidders' tradeoffs among relevant packages and promotes efficient outcomes.

Revealed preference constraints that are not needed to prevent undesirable behavior are not included. This simplifies the activity rule and gives the bidders greater flexibility throughout the auction. Supplementary bids are only constrained by revealed preference with respect to the final round and relevant rounds in which the bidder reduced eligibility.

In the clock stage, the bidder is always able to place bids that are consistent with eligibility point monotonicity. This provides one easy way for the bidder to see how a package can be modified to satisfy the activity rule. The eligibility-point safe harbor also provides additional flexibility in the event that a bidder's values change during the clock stage as a result of price discovery.

## 3. Technical Description

***Eligibility Points of Products***: Each Product is assigned a number of Eligibility Points. This is expressed as a vector, *e*, of Eligibility Points for each Product.

***Eligibility Points of a Package or a Bid***: The Eligibility Points of a Package *q* or of a Bid is the dot product of the quantity vector of Products and the vector of Eligibility Points for each Product, i.e. $E(q) = e \cdot q$ .

***Eligibility*** of a Bidder in the initial Clock Round is based on their Financial Deposit (so is exogenous). In each subsequent Clock Round, a Bidder's Eligibility equals the lesser of its Eligibility in the previous round and the Eligibility Points of its Package bid in the previous round, i.e. $E_t^i = \min\{E_{t-1}^i, E(q_{t-1}^i)\}$ . Eligibility can never increase in later rounds.

Let *f* denote the final Clock Round.

An ***Eligibility-Reducing Round*** for Bidder *i* is any Clock Round *t* in which Bidder *i*'s Eligibility is reduced, i.e. any $t\ (1 \le t \le f)$ such that $e \cdot q_t^i < E_t^i$.

Let $\mathcal{E}^i$ denote the set of Eligibility-Reducing Rounds for Bidder *i*, i.e. $\mathcal{E}^i = \left\{ t : 1 \le t \le f \text{ and } e \cdot q_t^i < E_t^i \right\}$.

For any Bidder *i* and any Package *q* within Bidder *i*'s initial Eligibility, let $t^i(q)$ denote the last Clock Round in which Bidder *i* was eligible to bid for Package *q*, i.e. $t^i(q) = \max\left\{ t \le f : e \cdot q \le E_t^i \right\}$.

Observe that $t^i(q) \in \mathcal{E}^i \cup \{f\}$.

For any Package *q*, define $\mathcal{E}^i(q)$ to be the set $\mathcal{E}^i \cup \{f\}$ truncated to exclude all rounds prior to $t^i(q)$, i.e. $\mathcal{E}^i(q) = \left\{ t \in \mathcal{E}^i \cup \{f\} : t \ge t^i(q) \right\}$.

Let $p_t$ denote the Clock Prices in Clock Round *t*. Let the implied price of Package *q* in Clock Round *t* be denoted by: $A_t(q) = p_t \cdot q$.

Let $b^i(q)$ denote a bound (which we will now develop) that Bidder *i* may submit for Package *q* in the Supplementary Bid Round.

Also, for Package $q_t$ (Bidder *i*'s package bid in Round *t*), let $B^i(q_t)$ denote the maximal bid that Bidder *i* has submitted for Package $q_t$ in any Clock Round or the Supplementary Round.

If in Clock Round *t*, Bidder *i* bid on Package $q_t$ when she could have instead bid on Package *q* at an implied price of $p_t \cdot q$, revealed preference would tell us that:

$$v^i(q_t) - p_t \cdot q_t \ge v^i(q) - p_t \cdot q .$$

We do not know the value $v^i(q_t)$; in place of the value, we use $B^i(q_t)$. Similarly, we substitute $b^i(q)$ in place of $v^i(q)$. Making these substitutions and rearranging terms gives us a Revealed-Preference Constraint on Bidder *i* for Package *q* with respect to Round *t*, which we denote RP(*i*,*q*,*t*):

(RP(*i*,*q*,*t*))
$$b^i(q) \le B^i(q_t) + p_t \cdot (q - q_t) .$$

## 4. Proposed Activity Rule for the Clock Rounds

### 4.1 Statement of Eligibility-Point Monotonicity

Bidder *i* may bid on Package $q_t$ in Clock Round *t* if and only if:

(E-Pt Monotonicity) $e \cdot q_t \leq E_t^i$.

### 4.2 Statement of the Proposed Alternative

**Short name: Simplified RP/Eligibility-Point Hybrid**

**Long name: Simplified Revealed Preference with an Eligibility-Point Safe Harbor**

Bidder *i* may bid on Package $q_t$ in Clock Round *t* if:

(Simplified RP) $(p_t - p_s) \cdot (q_t - q_s) \leq 0$, for all $s < t$ such that $s \in \mathcal{E}^i(q)$,

***or*** if:

(E-Pt Monotonicity) $e \cdot q_t \leq E_t^i$.

Simplified RP is the standard textbook revealed-preference constraint with linear prices, but imposed only in eligibility-reducing rounds. The derivation is standard: If in Clock Round *s*, Bidder *i* bid on Package $q_s$ when she could have instead bid on Package $q_t$, it is the case that $v^i(q_s) - p_s \cdot q_s \geq v^i(q_t) - p_s \cdot q_t$. Meanwhile, if in Clock Round *t*, Bidder *i* bid on Package $q_t$ when she could have instead bid on Package $q_s$, it is the case that $v^i(q_t) - p_t \cdot q_t \geq v^i(q_s) - p_t \cdot q_s$. Adding these two inequalities and rearranging the terms yields $(p_t - p_s) \cdot (q_t - q_s) \leq 0$. Package $q_t$ has become relatively less expensive than $q_s$.

There is sometimes a concern that, if a bidder fails to bid according to a consistent set of valuations (or the bidder revises her valuations during the auction), she may find herself unable to find any bid that satisfies revealed preference. For that reason, we allow Eligibility-Point Monotonicity as a “safe harbor”. Even if there are no bids remaining that are consistent with the revealed-preference constraint, the bidder is still free to submit a bid for any Package whose number of Eligibility Points is within the bidder’s Eligibility.

## 5. Proposed Activity Rule for the Supplementary Bid Round

### 5.1 Statement of Ofcom’s (2011) “Relative Cap”

The bound on the bid that Bidder *i* may submit for Package *q* in the Supplementary Bid Round is given ***only*** by $RP(i, q, t^i(q))$. Consequently, the constraints are:

(Relative Cap) $b^i(q) \leq B^i(q_t) + p_t \cdot (q - q_t)$, for all packages *q* and for $t = t^i(q)$.

Moreover, the "iterative" calculations described in Ofcom's Relative Cap come directly from a chain of inequalities, one for each reduction in eligibility starting with round $t^i(q)$, that can be produced by the statement immediately above.

However, a problem with the Relative Cap is that it need not satisfy RP(*i*,*q*,*f*) for Packages *q* such that $t^i(q) < f$ ; we have simple counterexamples demonstrating this.

## 5.2 Statement of the Proposed "Simplified Revealed-Preference Cap"

The bound on the bid that Bidder *i* may submit for Package *q* in the Supplementary Round is given by $RP(i,q,t^i(q))$ ***and*** all subsequent rounds in $\mathcal{E}^i(q)$. (This includes all subsequent eligibility-reducing rounds, as well as the final clock round, *f*.) In addition, we make the revealed-preference inequality strict; observe that this may formally require increasing the bid on $q_f$ by one bidding unit—otherwise, bidder *i*'s past bids (as well as her implicit zero bid for the empty set) might only satisfy this constraint with equality, rather than with strict inequality. Consequently, the constraints are:

(Simplified RP Cap) $$b^i(q) < B^i(q_t) + p_t \cdot (q - q_t), \text{ for all packages } q \neq q_t \text{ and for all } t \in \mathcal{E}^i(q).$$

The difference between this formulation and Ofcom's Relative Cap is that RP(*i*,*q*,*t*) is required to be satisfied not only for $t^i(q)$ (the last Clock Round in which Bidder *i* was eligible to bid for Package *q*) but also for all later Eligibility-Reducing Rounds and the final Clock Round. In particular, RP(*i*,*q*,*f*) (the final round constraint) is required to hold for *all* Packages *q*, so if all items are allocated in the final Clock Round, then the allocation will be unchanged after the Supplementary Round.

Another alternative cap ("between" the Relative Cap and the Simplified RP Cap) that would be sufficient to assure that if all items are allocated in the final Clock Round, then the allocation will be unchanged after the Supplementary Round, is $b^i(q) \leq B^i(q_t) + p_t \cdot (q - q_t), \text{ for } t = t^i(q) \text{ and for } t = f$ . This addresses the flaw in the Relative Cap that it need not satisfy RP(*i*,*q*,*f*). The reason why we impose the stronger condition of the Simplified RP Cap is as follows: Consider any $t \in \mathcal{E}^i(q)$ such that $t^i(q) < t < f$ . If it had turned out that *t* was the final clock round, then we would have imposed, on bidder *i*'s supplementary bids, the constraint that $b^i(q) \leq B^i(q_t) + p_t \cdot (q - q_t)$ . By imposing $\text{RP}(i,q,t) \text{ for } t^i(q) < t < f$ , we are making the decision to impose this constraint irrespective of whether the clock stage closes in round *t* — that is, whatever consideration leads us to impose this constraint in the event that *t* is the final clock round should also lead us to impose the same constraint in the event that *t* is *not* the final clock round. This larger set of constraints further motivates bidders to bid on their most-preferred packages throughout the clock stage.

## 6. Results on the Simplified Revealed-Preference Cap

Cramton (2009) argued that a property of fundamental importance for the activity rule on supplementary bids to satisfy is a basic stability between the tentative allocation after the clock stage and the final allocation after the supplementary round. We will show that the strict version of RP(*i*,*q*,*f*) is sufficient to guarantee this stability. (Effectively, the strict version of RP(*i*,*q*,*f*) is the least binding cap in the spirit of revealed preference that implies Propositions 1, 2, 3 and 4, below.)

In this section, we write $q_f^i$ for Bidder *i*'s bid in the final clock round and we write $q^i$ for any possible package associated with Bidder *i*. Define:

(RP(*i*,*q*,*f*) Strict) $$b^i(q^i) < B^i(q_f^i) + p_f \cdot (q^i - q_f^i), \text{ for all } q^i \neq q_f^i \ .$$

We have:

**Proposition 1**. *If the clock stage ends with no excess supply and if supplementary bids are required to satisfy RP(i,q,f) Strict, then the final allocation is the same as the tentative allocation at the end of the clock stage.*

**Proof**. Consider any feasible allocation $\{q^i : i = 1, \ldots, n\}$. Observe, applying RP(*i*,*q*,*f*) Strict, that:

(1) $$\sum_{i=1}^n b^i(q^i) \leq \sum_{i=1}^n b^i(q_f^i) + p_f \cdot \left(\sum_{i=1}^n q^i - \bar{Q}\right),$$

where $\bar{Q}$ denotes the supply in the auction, and that:

(2) $$\sum_{i=1}^n b^i(q^i) = \sum_{i=1}^n b^i(q_f^i) + p_f \cdot \left(\sum_{i=1}^n q^i - \bar{Q}\right) \text{if and only if } q^i = q_f^i \text{ for all } i = 1, \ldots, n \ .$$

Moreover, for any feasible allocation, the second term of the right sides of (1) and (2) is at most zero. Consequently, $\sum_{i=1}^n b^i(q^i) < \sum_{i=1}^n b^i(q_f^i)$ for any allocation $q^i \neq q_f^i$, establishing that $q^i = q_f^i$ (i.e. the final allocation equals the allocation at the end of the clock stage). □

Since the Simplified RP Cap incorporates RP(*i*,*q*,*f*) Strict, this has the immediate corollary:

**Corollary 1′**. *If the clock stage ends with no excess supply and if supplementary bids are required to satisfy the Simplified RP Cap, then the final allocation is the same as the tentative allocation at the end of the clock stage.*

Alternatively, suppose that the clock stage ends with unallocated items, $q_u$. As first observed by Cramton (2009, Proposition 2), if any bidder *k* sets its supplementary bid for the package it demanded at the final clock prices by increasing its final clock bid by the value of the unallocated items (priced at the final prices), then bidder *k* guarantees itself winning every item that it demanded at the final clock prices:

**Proposition 2**. *If the clock stage ends with a quantity* $q_u$ *of unallocated items and with positive prices, and if supplementary bids are required to satisfy RP(i,q,f) Strict, then bidder k can guarantee itself winning every item that it demanded at the final clock prices by submitting a supplementary bid of* $b^k(q_f^k) = p_f \cdot (q_f^k + q_u)$ *and by not submitting any supplementary bids for packages not containing* $q_f^k$*.*

**Proof**. By allocating $\{q_f^i : i = 1, \ldots, n\}$ to each bidder, the auctioneer can assure a combined bid value of $\sum_{i=1}^n b^i(q_f^i)$. Meanwhile, applying RP(*i*,*q*,*f*) Strict, any feasible allocation $\{q^i : i = 1, \ldots, n\}$ cannot receive a combined value of greater than:

(3) $$\sum_{i=1}^n b^i(q^i) < \sum_{i=1}^n b^i(q_f^i) + p_f \cdot q_u \ ,$$

since at least one bidder *i* must win $q^i \neq q_f^i$ or else items go unallocated. Moreover, the combined bid value incurs a penalty of $p_f \cdot q_u$ in any allocation in which bidder *k* fails to win all of $q_f^k$, as compared to an allocation in which bidder *k* does win all of $q_f^k$, due to bidder *k*'s supplementary bid (and its lack of supplementary bids on packages not containing $q_f^k$). Using (3), if bidder *k* does not win all of $q_f^k$:

(4) $$\sum_{i=1}^n b^i(q^i) < \sum_{i=1}^n b^i(q_f^i) \ .$$

This establishes that bidder *k* wins all of $q_f^k$ in the final allocation. □

Continue to assume that the clock stage ends with unallocated items, $q_u$. If at least one bidder places the maximum allowable supplementary bid permitted by RP(*i*,*q*,*f*) Strict on the union of its clock allocation and the set of unallocated items, then the final allocation has each bidder *i* winning either package $q_f^i$ or package $q_f^i + q_u$, a result stronger than we found in Proposition 2:

**Proposition 3**. *If the clock stage ends at positive prices and with unallocated items, but if at least one bidder places a supplementary bid on the union of its clock allocation and the set of unallocated items at the maximum price permitted by RP(i,q,f) Strict, then the final allocation has one bidder i winning package* $q_f^i + q_u$ *and all other bidders i winning packages* $q_f^i$*.*

**Proof**. Consider any feasible allocation $\{q^i : i = 1, \ldots, n\}$. Observe, applying RP(*i*,*q*,*f*) Strict, that:

(5) $$\sum_{i=1}^n b^i(q^i) \leq \sum_{i=1}^n b^i(q_f^i) + p_f \cdot \sum_{i=1}^n \left(q^i - q_f^i\right) - 1 \leq \sum_{i=1}^n b^i(q_f^i) + p_f \cdot q_u - 1 \,,$$

since at least one bidder *i* must win $q^i \neq q_f^i$ or else items go unallocated. If at least one bidder (who will be denoted by *k*) places the supplementary bid $b^k(q_f^k + q_u) = b^k(q_f^k) + p_f \cdot q_u - 1$, then $\sum_{i=1}^n b^i(q^i)$ can attain the upper bound of $\sum_{i=1}^n b^i(q_f^i) + p_f \cdot q_u - 1$ via the allocation of $q^k = q_f^k + q_u$ and $q^i = q_f^i$, for $i \neq k$. Moreover, unless the final allocation has one bidder *i* winning package $q_f^i + q_u$ and all other bidders *i* winning packages $q_f^i$, there would be at least two bidders *i* not winning $q^i \neq q_f^i$, and so

$\sum_{i=1}^{n} b^i(q^i) \leq \sum_{i=1}^{n} b^i(q_f^i) + p_f \cdot q_u - 2$. Consequently, the final allocation has one bidder *i* winning package $q_f^i + q_u$ and all other bidders *i* winning packages $q_f^i$. □

Finally, note that the outcome of Proposition 3 is fairly severe in that the entire set of unallocated items is awarded to a single bidder. A slight modification to the above strict-inequality cap is, instead, to limit the incremental bid for any incremental set, *q*, of unallocated items to be no greater than $p_f \cdot q - \#(q)$, where $\#(q)$ denotes the number of items in the set *q*. Such a modification enables the feasibility of various unallocated items being allocated to different bidders. Moreover, we still have:

**Proposition 4**. *Suppose that the clock stage ends at positive prices and with unallocated items, and that the incremental bid for any incremental set, q, of unallocated items is limited to be no greater than* $p_f \cdot q - \#(q)$. *If at least one bidder k places a supplementary bid on the union of its clock allocation and the set of unallocated items at the maximum allowable price of* $b^k(q_f^k + q_u) = b^k(q_f^k) + p_f \cdot q_u - \#(q_u)$, *then the final allocation has every bidder winning every item that it demanded at the final clock prices.*

The propositions demonstrate that the clock stage provides excellent price and allocation discovery whenever the final clock allocation has little or no excess supply. Winners in the final clock allocation know how to guarantee winning at least their final clock allocations. It is not necessary to increase bids to full value. A clock winner only needs to raise its final clock bid by the value of the unallocated items, evaluated at the final clock prices. Potential clock losers have an incentive to bid until no profitable packages remain, since losing in the clock stage may prevent them from winning any items.

## 7. Conclusion

One of the great virtues of the simultaneous multiple-round auction (with bids on individual items) is that each bidder's winnings are based on incremental decisions to bid higher. A bidder can always guarantee winning a particular package by continuing to bid on each item in the package until all rivals are pushed aside. This incremental bidding limits what winners must disclose about their values. And losers know why they lost. However, complementarities may force the winner to pay more than its value in this process and therefore a rational bidder will sometimes drop out before full value is reached. Package bids are used in the CCA to handle complementarities and address this problem. However, we would like to retain the benefits of incremental bidding to the extent possible.

The activity rules proposed here are intended to retain the key benefits of price and allocation discovery seen in the SMRA. Despite allowing a final sealed-bid round (the supplementary round), the winner determination is far from a mysterious black box. The clock stage reveals much about the final allocation. In the simplest case where all items are allocated at the end of the clock stage, the tentative allocation is final. In other cases, winners know how to guarantee that they will win at least the clock allocation.

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