

Fear of Losing in Dynamic Auctions: An Experimental Study

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Abstract

We analyze the implications of different pricing rules in discrete clock auctions. The two most common pricing rules are highest-rejected bid (HRB) and lowest-accepted bid (LAB). Under HRB, the winners pay the *lowest* price that clears the market; under LAB, the winners pay the *highest* price that clears the market. This pricing difference creates stronger incentives for bid shading under LAB. When bidders seek to maximize profits, the HRB auction maximizes revenues and is fully efficient. Because of the bid shading under LAB, a bidder may lose at an affordable price. Bidders who fear losing may limit bid shading, causing the LAB auction to achieve higher revenues than the HRB auction. Our experiments confirm that this is the case. The LAB auction achieves higher revenues. This also is the case in a version of the clock auction with provisional winners that is commonly used in spectrum auctions. This revenue result may explain the frequent use of LAB pricing despite the efficiency and simplicity advantages of HRB pricing.

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1 Introduction

A common method to auction radio spectrum, electricity, gas, and other products is the discrete clock auction. The auctioneer names a price and each bidder responds with her desired

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quantity. If there is excess demand, the auctioneer then names a higher price. The process continues until there is no excess demand.

Discrete rounds are used in practice to simplify communication, make the process robust to communication failures, and mitigate tacit collusion (Ausubel and Cramton 2004). An implication of discrete rounds is that the pricing rule matters. The two most common pricing rules are lowest-accepted bid and highest-rejected bid. Another issue is whether the bidder can specify an exit bid—a price less than the current price at which the bidder desires to reduce quantity. In the limit as the size of the bid increment goes to zero, the distinction between pricing rules is irrelevant and exit bids are unnecessary. However, in practical auctions where the number of rounds often ranges from 4 to 10, discreteness matters.

We examine bidding behavior under three versions of a discrete clock auction. In each version, after each round the bidders learn the aggregate demand. To prevent bid-sniping, an activity rule requires that a bidder's quantity demanded cannot increase at higher prices. Bidders can only maintain or reduce quantity as the price rises. The three versions differ in the pricing rule and whether bidders make exit bids to express the price at which a quantity reduction is desired.

Highest-rejected bid (HRB). If the bidder reduces quantity in a round, the bidder names a price for each quantity reduction. The price of each reduction must be greater than the prior price and less than or equal to the current price. Each exit price is interpreted as the price at which the bidder is indifferent between the higher quantity and the lower quantity. If there is no excess demand at the current price, the supply is awarded to the highest bidders, and each winner pays the highest-rejected bid for the quantity won. The clearing price is the *lowest* price consistent with market clearing—the price at which supply equals demand.

Lowest-accepted bid (LAB). This is the same as HRB, except that the winners pay the lowest-accepted bid for the quantity won. The clearing price is the *highest* price consistent with market clearing.

Lowest-accepted bid with provisional winners (LABpw). This is the same as LAB, except there are no exit bids. Instead after each round, provisional winners are determined. Those with

the highest price bid are selected first, and, in the event of a tie, the remaining provisional winners are selected at random.

The clock auction is best thought of as a dynamic version of a sealed-bid uniform-price auction. In the uniform-price auction, the auctioneer collects a demand curve from each bidder, forms the aggregate demand curve, and crosses it with the supply curve to determine the market clearing price and the quantity won by each bidder. The clock auction does the same thing, but gathers the demand curves from each bidder in a sequence of discrete rounds, and bidders receive information about excess demand at the end of each round. The uniform-price auction is just a single-round clock auction. In both clock auctions and uniform-price auctions, two pricing rules are commonly used: highest-rejected bid and lowest-accepted bid. This is the motivation for our HRB and LAB treatments.

Our third treatment, lowest-accepted bid with provisional winners, is a version of the simultaneous ascending auction commonly used to auction radio spectrum. The government of India has proposed this format for its 3G spectrum auction. Similar approaches have been used elsewhere, such as in spectrum auctions in Canada and Italy.

A common motivation for using lowest-accepted bid, rather than highest-rejected bid, is higher revenues. At first glance, it would seem that selecting the highest clearing price (LAB) would result in greater revenue than selecting the lowest clearing price (HRB). The argument is incomplete, since the pricing rule influences behavior. LAB provides a stronger incentive for shading one's bid below value. In simple cases, the greater bid shading under LAB exactly offsets the revenue gain from selecting the higher clearing price. Revenue equivalence obtains, and the two pricing rules result in the same expected revenue—at least in theory when bidders seek to maximize profits.

Bid shading in LAB may lead to some situations where a bidder finds herself losing at an affordable price. For example, a bidder who values the auctioned good at \$84 and exits at price level \$70 with an exit bid of \$65 will regret her bid if she finds out that the winning bidder exits at this price level with an exit bid of \$67—she could have made a profit with an exit bid of more than \$67. There can be other situations where a bidder loses the object at an affordable price but it is not clear how the outcome would change if she changed her action. In the example above, if

the bidder loses and finds out that her opponent won by staying in the auction at a price level of \$70, she cannot infer that she would have won if she stayed in as well. Instead she can at most imagine some events in which she would be better off if she did not exit at \$70. One of those events is that at a price level of \$70 both bidders stay in and at a price level of \$80 she exits with an exit bid of \$78 and her opponent exits with an exit bid of \$75. In such an event, the bidder would make positive profit. The bidder who fears losing and experiencing such situations as in the example above will shade her bid less than a bidder focused solely on profit maximization. LAB may achieve higher revenues if bidders fear losing.

This type of regret, loser regret, has been shown to explain overbidding in first-price sealed-bid auctions (Filiz-Ozbay and Ozbay 2007, Engelbrecht-Wiggans and Katok 2007, 2008). In a first-price auction if a bidder learns that the winning bid is less than her value, the ex-post best action is bidding a little bit more than the winning bid. Hence, the source of loser regret in first-price auctions is defined to be the difference between the bidder's value and the winning bid whenever the winning bid is affordable. In contrast to the first-price auction, in dynamic auctions it may be unclear for a losing bidder how the outcome would change if she bid differently. Therefore, ex-post a losing bidder may only regret in expectation. Kőszegi and Rabin (2006) provide a model of reference-dependent preferences and loss aversion² where the reference point can be stochastic (see also Sugden 2003). In our setting, the outcome of alternative actions of a bidder may serve as the stochastic reference point,³ and hence the loser regret in expectation as a source of fear of losing can be incorporated into dynamic auctions with the model of Kőszegi and Rabin (2006).

The purpose of this paper is to examine the bidding behavior and outcomes, especially efficiency and revenue, under the three different formats in the experimental lab. Our main hypothesis is that subjects will overbid under lowest-accepted bid, consistent with the fear of losing; whereas, under highest-rejected bid, bidders will bid truthfully. Thus, revenues under LAB will be higher than revenues under HRB. However, efficiency will be higher under HRB, as a result of the simpler bidding strategies and the absence of differential bid shading. Although

² Loss aversion has been experimentally studied in various papers (for example, Camerer 1995; Knetsch et al. 1991).

³ In the first-price sealed bid auctions, the winning bid is the reference point for modeling regret.

the formats apply to the general case of auctioning many units of multiple products, for simplicity we restrict attention to the case of auctioning a single good.

Our results are consistent with the fear-of-losing hypothesis. Revenues under both LAB and LABpw are significantly higher than under HRB. Thus, in settings where revenue is the predominant objective, the seller may favor LAB, but in settings where efficiency and simplicity are of greater concern, then the seller may favor HRB.

Many experimental papers have examined bidding behavior in sealed-bid auctions, and observed this tendency for overbidding in first-price auctions (see Cox, et al. 1982, 1988, as the seminal papers; Kagel 1995; Kagel and Levin 2008 for detailed surveys). Risk aversion offers one explanation, but this has proven inadequate (see Kagel 1995). Several papers explain the overbidding phenomena with behavioral motives (for example, Goeree et al. 2002, Crawford and Iriberry 2007; Filiz-Ozbay and Ozbay 2007; Lange and Ratan 2009). Delgado et al. (2008) provide a neurological foundation that fear of losing, not joy of winning, explains overbidding in first-price sealed-bid auctions. Reference dependent preferences in auctions are studied by Lange and Ratan (2009) in sealed bid auctions and by Shunda (2009) in auctions with buy-it-now prices. There are also a number of experimental papers that use clock auctions. Some of these papers use a continuous clock (e.g. Kagel and Levin 2001). Others use a discrete clock, and compare a sealed-bid auction with a particular discrete clock (e.g. Ausubel et al. 2009). This paper's contribution is understanding the implications of different pricing rules in discrete clock auctions.

In Section 2, we begin with a presentation of the theory for profit maximizing bidders. Equilibrium bidding strategies for the three versions of the discrete clock auction are characterized in Cramton and Sujarittanonta (2010). Here we summarize the results, as specialized to our experimental setting. In Section 3, we present the theory with fear of losing that is motivated by loser regret. The experimental design and the results are discussed in Sections 4 and 5, respectively. Section 6 concludes.

2 Theory with profit maximizing bidders

Two bidders $i \in \{1, 2\}$ compete to buy a single good. Bidder i 's private value for the good is v_i where each v_i is independently drawn from the uniform distribution on $[50, 100]$. Bidder i 's payoff if she wins the good at a price p is $v_i - p$. The seller values the good at 0.

Before the auction starts, the seller announces a vector of bid levels, $\mathbf{P} = (P_0, P_1, \dots, P_{T-1})$ where P_t is the price at round $t+1$ and T is the number of bid levels. The clock price increases every round so that $50 = P_0 < P_1 < \dots < P_{T-1} = 100$. The auction begins in round one at a price P_1 . For simplicity, we assume six bid levels, equally spaced: $\mathbf{P} = (50, 60, 70, 80, 90, 100)$.

In each round t , each bidder chooses either to bid at the current clock price or to exit. Once a bidder exits she cannot bid again. If both bidders stay in, the auction proceeds to the next round. If one bidder exits, then the bidder who stayed in wins the good. The resolution of the other cases and the payment rule will depend on the auction format.

2.1 Highest-rejected-bid (HRB)

In the HRB format, the final price is determined by the highest-rejected bid. If a bidder exits in round t , the bidder submits an exit bid—a price between P_{t-1} and P_t at which she wants to exit. Specifically, if one bidder exits, the remaining bidder wins and pays the exit bid of the bidder exiting; if both bidders exit, the bidder with the highest exit bid wins and pays the lowest exit bid.

Proposition 1. *In the HRB auction, truthful bidding (bidding up to one's valuation) is a weakly-dominant strategy. The HRB auction is efficient and maximizes seller revenue.*

The dominant strategy result is standard and holds regardless of the number of bidders, the number of goods, or how valuations are drawn. All that is required is that each bidder demands only a single good. Thus, highest-rejected bid is the Vickrey price, thereby inducing truthful bidding. Efficiency is an immediate implication of truthful bidding. The revenue maximization result follows from ex ante symmetry (the seller cannot gain from misassigning the good) and the fact that the lowest bidder valuation (50) is sufficiently above 0 that the seller cannot gain

from withholding the good (setting a reserve price above 50). For the uniform distribution, the marginal revenue of awarding the good to a bidder with value v is $2v - 100$, which is strictly increasing and nonnegative for all $v \geq 50$. Thus, the optimal auction awards the good to the bidder with the highest value.

The HRB auction has extremely desirable properties in our setting. It is both efficient and maximizes seller revenues. Moreover, the bidding strategy is simple—just bid up to your true value—and is best regardless of what the other bidder is doing. Another important property of the HRB auction is that a bidder cannot lose at an affordable price as long as she bids her value. Hence, neither the winner nor the loser ever regrets having bid as they bid. The winner could not do better by exiting earlier; the loser could not do better by staying in longer.

2.2 *Lowest-accepted-bid (LAB)*

In the LAB format, the final price is determined by the lowest-accepted bid. If a bidder exits in round t , the bidder submits an exit bid—a price between P_{t-1} and P_t at which she wants to exit. Specifically, if one bidder exits in a round, the remaining bidder wins and pays the current price; if both bidders exit in a round, the bidder with the highest exit bid wins and pays her exit bid.

Proposition 2. *In the symmetric equilibrium of the LAB auction (see Table 1), a bidder's exit bid $b(v_i)$ is strictly increasing in v_i and $b(v_i) < v_i$ for all $v_i > 50$. The LAB auction is efficient and maximizes seller revenue.*

Table 1. Equilibrium Strategy for LAB auction

Valuation	Exit Round	Exit bid function
$[50, 70)$	1	$25 + \frac{1}{2}v_i$
$[70, 90)$	3	$35 + \frac{1}{2}v_i$
$[90, 100]$	5	$45 + \frac{1}{2}v_i$

Ex ante symmetry—the fact that the bidders' values are drawn from the same distribution—is critical for Proposition 2. This allows for a symmetric equilibrium. Since the exit bid functions

are the same and strictly increasing, the bidder with the highest value wins and the outcome is efficient. Revenue maximization then follows from the revenue equivalence theorem. The assignment is the same as in the HRB auction, and both auctions give the bidder with a value of 50 a payoff of 0.

The LAB auction in our setting is efficient and maximizes seller revenues. Nonetheless, one might favor the HRB auction because of its simple bidding strategy without bid shading. In the LAB auction, bidders must do a difficult equilibrium calculation to determine the optimal level of bid shading. As a result of the bid shading, the loser experiences loser's regret whenever the winning price is below her value.

2.3 *Lowest-accepted-bid with provisional winners (LABpw)*

Our third version of the discrete clock auction has provisional winners instead of exit bids. This approach is common in spectrum auctions, and has been proposed for the India 3G spectrum auction.

One of the bidders is selected at random as the provisional winner at the starting price $P_0 = 50$. In each round, each bidder chooses either to bid at the current price or to exit. If both bidders stay in, one of the bidders is selected at random as the provisional winner and the auction proceeds to the next round. If one bidder exits, then the bidder who stayed in wins the good at the current price. If both bidders exit, then the provisional winner wins the good at the prior price.

An important difference between the auction described here and a standard ascending-bid auction is a provisional winner must keep topping her own bid in order to be eligible to bid in subsequent rounds. In contrast, in a standard ascending-bid auction, a provisional winner does not need to bid, since if her bid is topped, she can still bid in the next round.

Proposition 3. *In a perfect Bayesian equilibrium of the LABpw auction, the provisional loser stays in provided her value is not yet reached; the provisional winner exits at a level below her true value (see Table 2). The exit level depends on the provisional winning history. The outcome is inefficient and does not maximize seller revenue.*

Table 2. Provisional Winner’s Equilibrium Strategy for LABpw auction

Round	History[†]	Critical Valuation[‡]
1	(1)	74
2	(1,1)	84
	(0,1)	70
3	(1,1,1) and (1,0,1)	94
	(0,1,1)	90.8
	(0,0,1)	80
4	(1,0,0,1), (0,1,0,1) and (0,0,0,1)	90
	Otherwise	100
5	Any	100

[†]History vector denotes whether the bidder is a provisional winner (denoted by 1) or not (denoted by 0) in all the rounds up to the current round;

[‡]Critical Valuation denotes the threshold such that the bidder stays in if her value is above the corresponding threshold.

The LABpw auction uses provisional winners, rather than exit bids, to determine who wins in the event both bidders exit in the same round. This creates two sources of inefficiency. First, without exit bids, there is no precise value information to determine who has the higher value. Second, the provisional winner designation creates differential bid shading (provisional winners shade bids, whereas provisional losers do not). Since in our setting there is no conflict between efficiency and revenue maximization, it is clear that the LABpw auction does not maximize seller revenue. The seller should favor HRB or LAB on both efficiency and revenue grounds. In addition, the equilibrium strategies in LABpw are greatly complicated by history dependence.

3 Theory with fear of losing

One explanation for the common use of LAB pricing in practice is that it yields higher revenues than HRB pricing. Based on a theory with profit maximizing bidders such an explanation is questionable. HRB pricing does at least as well as both LAB and LABpw in terms of both efficiency and revenues. Moreover, HRB simplifies bidding, since truthful bidding is a weakly-dominant strategy. The revenue explanation for favoring LAB pricing must rest on the view that bidders systematically deviate from the equilibrium behavior described in the prior

section. For example, bidders faced with LAB pricing systematically engage in too little bid shading. A behavioral explanation consistent with this hypothesis is that bidders anticipate losing at a profitable price, and in order to minimize this they engage in less bid shading. We now extend the theory to allow this possibility.

Filiz-Ozbay and Ozbay (2007) showed that in first-price sealed-bid auctions, the bidders who receive feedback on the winning price anticipate feeling a negative emotion (loser's regret) if the price is below their values and reflect this anticipation to their bids by bidding more aggressively. In loser's regret, bidders take an affordable winning price (outcome of ex-post best action) as a reference point in their utility calculations. This theory requires disclosure of the reference point to the bidders which is natural in the first-price setting. However, in a dynamic auction this may not be the case. For example, with LAB pricing, if a bidder with valuation \$90 exits when the clock is \$70 and observes that the other bidder did not exit in this round, then she will regret exiting too early. However, it is not certain that if she stayed in, she would have won the auction. After observing that the opponent stayed in, the losing bidder can only think how the resolution of the auction would be in expectation if she changed her bid in the last round. This type of bidder takes the ex-post best outcome as reference and compares her actual payoff with the payoff of this reference point. In the dynamic setting, the reference point (the outcome of ex-post best action) is stochastic. Kőszegi and Rabin (2006) provides a utility representation which allows for stochastic reference points. As in Kőszegi and Rabin (2006), we define the utility of a bidder when the auction outcome is c and the reference outcome is r as

$$u(c|r) = m(c) - \alpha \max\{0, m(r) - m(c)\}$$

where m is the consumption utility. The first term captures the monetary utility of the outcome. It is $v_i - p$ if bidder i wins at price p and zero otherwise. The second term compares the actual monetary utility with the reference monetary utility and if the latter is higher, the difference is a disutility. $\alpha \geq 0$ is bidder's fear of losing coefficient which can be motivated by regret in our setting. Any bid history in a dynamic auction generates the distribution (F) of consumption level and the distribution (G) of the reference point. Hence, as in Kőszegi and Rabin (2006), the expected utility of a bidder after any history can be represented as:

$$U(F|G) = \int \int u(c|r) dG(r) dF(c) \tag{1}$$

Filiz-Ozbay and Ozbay (2007) show that only the losers but not the winners engage in negative emotion. This means that, for example in LAB pricing, a bidder does not anticipate to feel bad if she wins with a high exit bid and learns that the exit bid of the opponent was much below hers. Therefore, in the formulation above, the reference outcome for a winner will be the outcome of the auction and the utility will be only the consumption utility.

Let $q_{it} \in \{0,1\}$ denote the bid of bidder i in round t where 0 means “exiting” and 1 means “staying in” the auction.

3.1 Highest-rejected-bid

In the HRB auction, when bidder i with $v_i \in [P_t, P_{t+1})$ considers staying in in round $t < I$, her expected utility is $\int_{z_t}^{b_t^{-1}(\alpha, P_t)} (v_i - b_t(\alpha, s)) dF(s | z_t) + \Pi_{t+1}(v_i)(1 - F(P_t | z_t))$ where $b_t(\alpha, s)$ is her opponent’s bidding function in round t given fear of losing coefficient α and valuation s , z_t is an inferred lower bound of the opponent’s valuation in round t , $F(s | z_t)$ is a conditional distribution function of valuation given that the lower bound of valuation is z_t and $\Pi_{t+1}(v_i)$ is the expected payoff if the auction continues to round $t+1$. When she considers staying out in round t , Equation (1) becomes

$$\pi_t(\alpha, v_i, B, z_t) = \int_{z_t}^{b_t^{-1}(\alpha, B)} (v_i - b_t(\alpha, s)) dF(s | z_t) - \alpha \max \left\{ 0, \int_{b_t^{-1}(\alpha, B)}^{b_t^{-1}(\alpha, P_t)} (v_i - b_t(\alpha, s)) dF(s | z_t) \right\} \\ - \alpha \sum_{k=t+1}^I \max \left\{ 0, \int_{b_k^{-1}(\alpha, P_{k-1})}^{b_k^{-1}(\alpha, P_k)} (v_i - b_k(\alpha, s)) dF(s | z_k) \right\} - \alpha \max \left\{ 0, \int_{b_{t+1}^{-1}(\alpha, P_t)}^{b_{t+1}^{-1}(\alpha, v_i)} (v_i - b_{t+1}(\alpha, s)) dF(s | z_{t+1}) \right\}$$

where B is bidder i ’s bid.⁴ The first term is the expected payoff by bidding B given that the opponent submits an exit bid below bidder i ’s bid and that the opponent’s value is at least z_t . The remaining terms are expected disutilities. The second term is for the event that both bidders exit at P_t but the opponent wins with a higher exit bid. The third term is for the events that bidder

⁴ When $v_i \in [P_{t-1}, P_t)$, her payoff of staying out in round t is

$$\pi_t(\alpha, v_i, B, z_t) = \int_{z_t}^{b_t^{-1}(\alpha, B)} (v_i - b_t(\alpha, s)) dF(s | z_t) - \alpha \max \left\{ 0, \int_{b_t^{-1}(\alpha, B)}^{b_t^{-1}(\alpha, v_i)} (v_i - b_t(\alpha, s)) dF(s | z_t) \right\}$$

i exits at P_t and the opponent stays in for the price levels less than P_t . The fourth term is for the event that the opponent exits at P_t but submits an exit bid that is lower than v_i . For example, consider bidder i with a valuation of \$84 exiting at the price level \$70 and submitting an exit bid of \$67. There are 3 possible events in which she expects to lose at an affordable level: (i) her opponent exits at \$70 but submits an exit bid higher than \$67, (ii) her opponent stays in at \$70 and exits at \$80, (iii) her opponent stays in until \$90 and exits at \$90 but submits a bid less than \$84. In each case, bidder i loses at an affordable level, and hence the difference between her value, \$84, and the exit bid of her opponent will be the source of the disutility.

Although there are possibilities for a bidder to regret her bid, in the equilibrium there is no incentive for bid shading in the HRB auction, in other words she bids her value regardless of α . z_t is therefore equal to P_t . Hence, the outcome remains the same as in Proposition 1.

Proposition 4. *In the HRB auction with fear of losing, truthful bidding (bidding up to one's valuation) is a weakly-dominant strategy. The HRB auction is efficient.*

3.2 Lowest-accepted-bid

In the LAB auction, when bidder i with $v_i \in [P_t, P_{t+1})$ considers staying in in round $t < I$, her expected utility is $\int_{z_t}^{b_t^{-1}(\alpha, B)} (v_i - B) dF(s|z_t)$ where B is bidder i 's bid, $b_t(\alpha, s)$ is her opponent's bidding function in round t given loss aversion coefficient α and valuation s , z_t is an inferred lower bound of the opponent's valuation in round t and $F(s|z_t)$ is a conditional distribution function of valuation given that the lower bound of valuation is z_t . When she considers exiting in round t , Equation (1) becomes⁵

⁵ When $v_i \in [P_{t-1}, P_t)$, her payoff of staying out in round t is

$$\pi_t(\alpha, v_i, B, z_t) = \int_{z_t}^{b_t^{-1}(\alpha, B)} (v_i - B) dF(s|z_t) - \alpha \max \left\{ 0, \int_{b_t^{-1}(\alpha, B)}^{b_t^{-1}(\alpha, v_i)} (v_i - b_t(\alpha, s)) dF(s|z_t) \right\}$$

$$\begin{aligned}
\pi_i(\alpha, v_i, B, z_i) = & \int_{z_i}^{b_i^{-1}(\alpha, B)} (v_i - B) dF(s | z_i) - \alpha \max \left\{ 0, \int_{b_i^{-1}(\alpha, B)}^{b_i^{-1}(\alpha, P_i)} (v_i - b_i(\alpha, s)) dF(s | z_i) \right\} \\
& - \alpha \sum_{k=i+1}^I \max \left\{ 0, \int_{b_k^{-1}(\alpha, P_{k-1})}^{b_k^{-1}(\alpha, P_k)} (v_i - b_k(\alpha, s)) dF(s | z_k) \right\} \\
& - \alpha \max \left\{ 0, \int_{b_{i+1}^{-1}(\alpha, P_i)}^{b_{i+1}^{-1}(\alpha, v_i)} (v_i - b_{i+1}(\alpha, s)) dF(s | z_{i+1}) \right\}
\end{aligned}$$

The first term is the expected payoff by bidding B given that the opponent exits and submits an exit bid below bidder i 's bid and that the opponent's value is at least z_i . The remaining terms are expected disutilities. The second term is for the event that both bidders stay out at P_i but the opponent wins with a higher exit bid. The third term is for the events that bidder i exits at P_i and the opponent stays in for the price levels less than P_i . The fourth term is for the event that the opponent exits at P_i but submits an exit bid that is lower than v_i . For example, consider bidder i with a valuation of \$84 exiting at the price level \$70 and submitting an exit bid of \$67. There are 3 possible events in which she expects to lose at an affordable level: (i) her opponent exits at \$70 but submits an exit bid higher than \$67, (ii) her opponent stays in at \$70 and exits at \$80, (iii) her opponent stays in until \$90 and exits at \$90 but submits a bid less than \$84. In each case, bidder i loses at an affordable level, and hence the difference between her value, \$84, and the exit bid of her opponent will be the source of the disutility.

Unlike the HRB auction, fear of losing weakens the incentive to bid shade with LAB pricing. Thus, for $\alpha > 0$, bidding is more aggressive than with $\alpha = 0$ implying higher revenues than under HRB pricing.

Proposition 5. *In the symmetric equilibrium of the LAB auction with fear of losing, a bidder's exit bid $b(\alpha, v_i)$ (see Table 3) is strictly increasing in α and v_i and $b(\alpha, v_i) < v_i$ for all $v_i > 50$ and $\alpha \geq 0$. The LAB auction is efficient. Seller revenues are strictly increasing in α . For $\alpha > 0$, LAB revenues are greater than HRB revenues.*

Table 3. Equilibrium Strategy for LAB auction with $\alpha \geq 0$

Valuation	Exit Round	Exit bid function
$\left[50, 50 + 10 \cdot \frac{2+\alpha}{1+\alpha} \right)$	1	$\frac{50}{2+\alpha} + \frac{1+\alpha}{2+\alpha} v_i$
$\left[50 + 10 \cdot \frac{2+\alpha}{1+\alpha}, 50 + 10 \cdot \frac{(2+\alpha)(1+2\alpha)}{(1+\alpha)^2} \right)$	2	$\frac{50}{2+\alpha} + 10 \cdot \frac{1}{1+\alpha} + \frac{1+\alpha}{2+\alpha} v_i$
$\left[50 + 10 \cdot \frac{(2+\alpha)(1+2\alpha)}{(1+\alpha)^2}, 50 + 10 \cdot \frac{(2+\alpha)(2+\alpha(4+3\alpha))}{(1+\alpha)^3} \right)$	3	$\frac{50}{2+\alpha} + 10 \cdot \frac{1+2\alpha}{(1+\alpha)^2} + \frac{1+\alpha}{2+\alpha} v_i$
$\left[50 + 10 \cdot \frac{(2+\alpha)(2+\alpha(4+3\alpha))}{(1+\alpha)^3}, 50 + 10 \cdot \frac{(2+\alpha)(2+\alpha(8+\alpha(9+4)))}{(1+\alpha)^4} \right)$	4	$\frac{50}{2+\alpha} + 10 \cdot \frac{2+\alpha(4+3\alpha)}{(1+\alpha)^3} + \frac{1+\alpha}{2+\alpha} v_i$
$\left[50 + 10 \cdot \frac{(2+\alpha)(2+\alpha(8+\alpha(9+4)))}{(1+\alpha)^4}, 100 \right]$	5	$\frac{50}{2+\alpha} + 10 \cdot \frac{2+\alpha(8+\alpha(9+4\alpha))}{(1+\alpha)^4} + \frac{1+\alpha}{2+\alpha} v_i$

3.3 Lowest-accepted-bid with provisional winners

We use the same approach for equilibrium characterization as Cramton and Sujarittanonta (2009). Let $\Pi_{i,t+1}(v_i, \mathbf{H}_{it}, \alpha)$ be the bidder i 's expected utility if the auction continues to round $t+1$ where \mathbf{H}_{it} is a vector of bidder i 's ranking history from round one to round t . Let $\hat{v}_t(\mathbf{H}_{it}, \alpha)$ be a critical valuation such that a bidder with valuation in $[\hat{v}_t(\mathbf{H}_{it}, \alpha), 100]$ bids in round t given a ranking history \mathbf{H}_{it} and a fear of losing coefficient α . An equilibrium bidding strategy is specified by critical valuations $\hat{x}_t(\mathbf{H}_{it}, \alpha)$ for all rounds $t=1, \dots, 5$ and for all possible ranking histories in Table 4. In the LABpw auction, when bidder i with v_i considers staying in in round t , Equation (1) becomes

$$\pi_{it}(v_i, \mathbf{H}_{it}, \alpha) = (v_i - P_t) \frac{P_t - \hat{v}_t(\mathbf{H}_{it}, \alpha)}{1 - \hat{v}_t(\mathbf{H}_{it}, \alpha)} + \Pi_{i,t+1}(v_i, \mathbf{H}_{it}, \alpha) \frac{1 - P_t}{1 - \hat{v}_t(\mathbf{H}_{it}, \alpha)}$$

where the first term is the expected payoff if the opponent exits in round t , and the second term is the expected payoff if the opponent stays in in round t .

When bidder i with v_i considers staying out in round t , Equation (1) becomes

$$\pi_{it}(v_i, \mathbf{H}_{it}, \alpha) = (v_i - P_{t-1}) \frac{P_t - \hat{v}_t(\mathbf{H}_{it}, \alpha)}{1 - \hat{v}_t(\mathbf{H}_{it}, \alpha)} - \alpha \max \left\{ 0, \Pi_{i,t+1}(v_i, \mathbf{H}_{it}, \alpha) \frac{1 - P_t}{1 - \hat{v}_t(\mathbf{H}_{it}, \alpha)} \right\}$$

where the first term is the expected payoff if the opponent exits in round t , and the second term is the expected disutility if the opponent stays in in round t . For example, consider bidder i , the provisional winner at price level \$70, with a valuation of \$84 exiting at \$70. If her opponent stays in at \$70 or at \$80 and if bidder i loses at \$80, bidder i will lose at an affordable level. Her expected payoff will be the source of the disutility.

Fear of losing has a similar effect as in the LAB auction, in particular the provisional winners engage in less bid shading.

Proposition 6. *In a perfect Bayesian equilibrium of the LABpw auction with fear of losing, the provisional loser stays in provided her value is not yet reached; the provisional winner exits at a level below her true value (see Table 4). The exit level depends on the provisional winning history. The outcome is inefficient. Seller revenues are strictly increasing in α . LABpw revenues are greater than HRB revenues for sufficiently large α .*

Table 4. Equilibrium Strategy for LABpw auction with $\alpha \geq 0$

Round	History [†]	Critical valuation [‡]	
		$0 \leq \alpha < 1$	$\alpha \geq 1$
1	(1)	$70 + 4 \cdot \frac{1-\alpha}{1+\alpha}$	$60 + \frac{20}{1+\alpha}$
2	(1,1)	$80 + 4 \cdot \frac{1-\alpha}{1+\alpha}$	$70 + \frac{20}{1+\alpha}$
	(0,1)	70	$80 + \frac{20(\alpha-1)}{(1+\alpha)^2}$
3	(1,1,1)	$90 + 4 \cdot \frac{1-\alpha}{1+\alpha}$	$80 + \frac{20}{1+\alpha}$
	(1,0,1)	$90 + 4 \cdot \frac{1-\alpha}{1+\alpha}$	$80 + \frac{20(3+\alpha^2)}{(1+\alpha)^3}$
	(0,1,1)	$90 + \frac{4(1+5\alpha)(1-\alpha)}{5(1+\alpha)^2}$	$80 + \frac{20}{1+\alpha}$
	(0,0,1)	80	$80 + \frac{20(\alpha-1)}{(1+\alpha)^2}$
4	(1,1,1,1)	100	$90 + \frac{20}{1+\alpha}$
	(1,1,0,1)	100	$90 + \frac{20(3+\alpha^2)}{(1+\alpha)^3}$
	(1,0,1,1)	100	$90 + \frac{20}{1+\alpha}$
	(1,0,0,1)	90	$90 + \frac{20(\alpha-1)}{(1+\alpha)^2}$
	(0,1,1,1)	100	$90 + \frac{20}{1+\alpha}$
	(0,1,0,1)	90	$90 + \frac{20(\alpha(3+\alpha+\alpha^2)-5)}{(1+\alpha)^4}$
	(0,0,1,1)	100	$90 + \frac{20}{1+\alpha}$
	(0,0,0,1)	90	$90 + \frac{20(\alpha-1)}{(1+\alpha)^2}$
5	Any	100	100

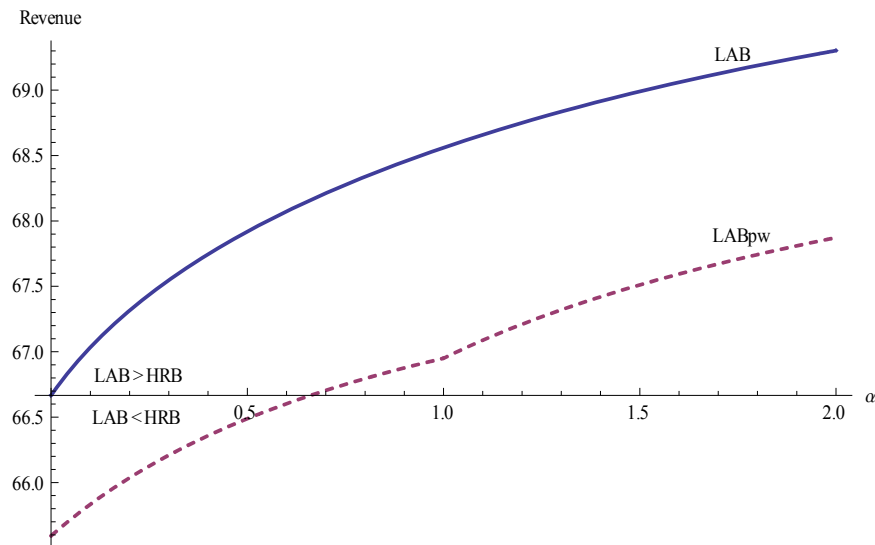
[†]History vector denotes whether the bidder is a provisional winner (1) or not (0) in all the rounds up to the current round;

[‡]Critical Valuation denotes the threshold such that the bidder stays in if her value is above the corresponding threshold.

3.4 Revenue Comparison

Figure 1 shows how revenue depends on the fear of losing coefficient α in our example with six bid levels. HRB revenue is 66.67 for all α , which is why the x-axis is set at that level. LAB revenue dominates HRB for all $\alpha > 0$. The LABpw revenues are lower than HRB revenues for small α , but are higher than HRB revenues for large α .

Figure 1. Revenue comparison as a function of α



4 Experimental method

The experiments were run at the Experimental Economics Lab at the University of Maryland. All participants were undergraduate students at the University of Maryland. The experiment involved six sessions. In each session one of the three treatments was administered. The numbers of participants in each treatment were 30 (HRB), 32 (LAB), and 30 (LABpw). No subject participated in more than one session. Participants were seated in isolated booths. Each session lasted about 80 minutes. Bidder instructions for each treatment are in the Appendix. To test each subject's understanding of the instructions, the subject had to answer a sequence of multiple choice questions. The auctions did not begin until the subject answered all of the multiple choice questions correctly.

In each session, each subject participated in 21 auctions. The first auction was a practice auction. Each auction had two bidders, selected at random among the subjects. Bidders were

randomly rematched after each auction. All bidding was anonymous. Bids were entered via computer. The experiment is programmed in z-Tree (Fishbacher 2007). At the conclusion of each auction, the bidder learned whether she won and the price paid by the winning bidder.

Bidders had independent private values for a fictitious good. All values were uniformly distributed between 50 and 100, rounded to the nearest cent. Both bidders were IN at the starting price of 50. The price increased by 10 if both bidders stayed IN the prior round. Thus, the possible price levels in each auction were 50, 60, 70, 80, 90, and 100. There were a maximum of five rounds in the discrete clock auction. The auction concluded as soon as a round was reached in which one or both bidders stayed OUT.

Treatment HRB. In each round, the computer asks the bidder if she is IN or OUT at the current price. If the bidder stays OUT, the bidder must specify an exit bid between the current price and the prior price. If the bidder stays IN and the opponent stays OUT, then the bidder wins and pays the opponent's exit bid. If the bidder stays OUT and the opponent stays IN, then the opponent wins at the bidder's exit bid. If both stay OUT, then the bidder with the higher exit bid wins and pays the smaller of the exit bids. If both stay IN, then the price increases by 10 and the auction continues to the next round.

Treatment LAB. In each round, the computer asks the bidder if she is IN or OUT at the current price. If the bidder stays OUT, the bidder must specify an exit bid between the current price and the prior price. If the bidder stays IN and the opponent stays OUT, then the bidder wins and pays her exit bid. If the bidder stays OUT and the opponent stays IN, then the opponent wins and pays her exit bid. If both stay OUT, then the bidder with the higher exit bid wins and pays her exit bid. If both stay IN, then the price increases by 10 and the auction continues to the next round.

Treatment LABpw. One of the bidders is selected at random as the provisional winner at the price of 50. In each round, the computer asks the bidder if she is IN or OUT at the current price. If the bidder stays IN and the opponent stays OUT, then the bidder wins and pays the current price. If the bidder stays OUT and the opponent stays IN, then the opponent wins at the current price. If both stay OUT, then the provisional winner of the prior round wins and pays the prior

round price. If both stay IN, then one of the bidders is selected at random as the provisional winner at the current price, price increases by 10 and the auction continues to the next round.

The winner in each auction earned her value minus the price paid in Experimental Currency Units (ECU). At the end of the experiment, total earnings were converted to US Dollars, at the conversion rate of 10 ECU = 1 US Dollar. Subjects also received a \$5 show-up fee. Cash payments were made at the conclusion of the experiment. The average subject payment was \$19.77.

5 Experimental results

Table shows the outcomes of each treatment. Treatment HRB, LAB and LABpw consist of 300, 320 and 300 auctions respectively. Treatment HRB and LAB, which are theoretically efficient, yield the efficient allocation with a frequency of 92% and 90.31%, respectively. Treatment LABpw yields the efficient allocation 85.33% of the time. Two sample t-tests are used to check for significant efficiency differences across different auction formats: LABpw is significantly less efficient than LAB ($t=1.89$, $p=0.03$) and HRB ($t=2.59$, $p=0.01$), but there is no significant difference between LAB and HRB ($t=0.74$, $p=0.46$).

To prevent misleading revenue results due to random variation of bidders' valuations across treatments, we will use seller's share of the gains from trade (the ratio of the price and winner's value) as the proxy for the revenues. Treatment HRB gives the seller a smaller share of the gains from trade than treatment LAB and LABpw. Kolmogorov-Smirnov test of seller's share distributions yielded significance difference between LAB and HRB ($p = 0.021$), and LABpw and HRB ($p = 0.001$), but there is no significant difference between LAB and LABpw ($p = 0.411$).

Table 5. Outcomes of treatment HRB, LAB and LABpw

	HRB	LAB	LABpw
Frequency of efficient allocation	92.00%	90.31%	85.33%
Seller's share of gains from trade	80.81%	83.79%	84.30%
Number of auctions	300	320	300

Figure 2. Actual, theoretical and Vickrey seller's share of gains from trade per auction

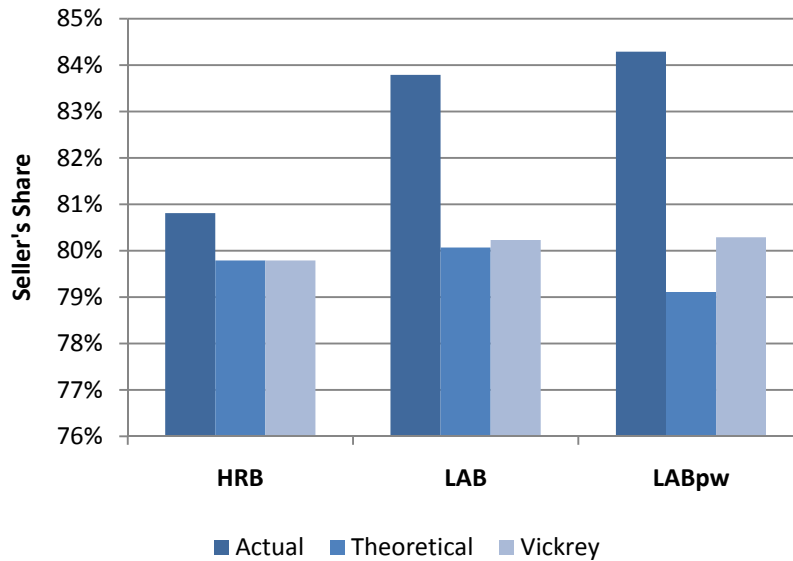


Figure 2 compares actual, theoretical and Vickrey seller's share of the gains from trade per auction—seller's share where Vickrey pricing is used instead of the corresponding pricing rule. Theoretical and Vickrey seller's share of HRB are equal because HRB is Vickrey pricing in a unit-demand setting. The actual seller's share of HRB is slightly higher than the theoretical one, but actual seller's share of LAB and LABpw are significantly higher than the theoretical prediction. This evidence implies that subjects bid more aggressively than the equilibrium prediction and as a result, the seller receives higher revenue. These aggressive bidding behaviors are investigated in the next subsections.

5.1 Bidding behavior in treatment HRB

Table 6. Summary statistics of selected variables in treatment HRB

Variable	Observations	Mean	Std.Dev.
Valuation	600	75.55	14.56
Valuation if exit bid is submitted	342	68.94	12.64
Exit bid	342	68.52	12.52

Table 6 shows the summary statistics. Subjects submitted a total of 342 exit bids. The deviation from truthful bidding is on average close to zero but there are instances of bid shading and bidding above one's valuation. 175 out of 342 exit bids are within one percent of the valuation.

Figure 3. Plot between valuations and exit bids in HRB auction

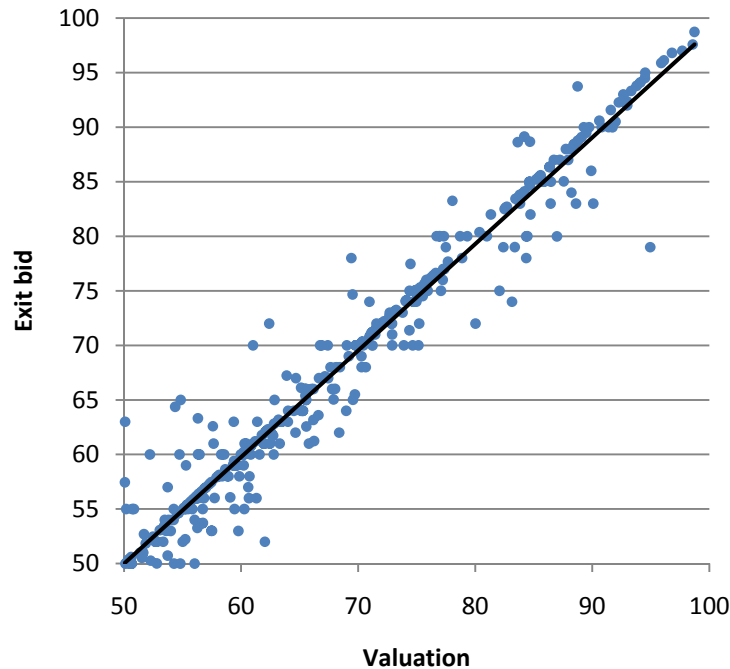


Figure 3 plots exit bids and corresponding valuations as well as their linear estimation. Since the data consists of roughly 10 exit bids from each subject, we run regression clustered by subjects to control the individual effects. The regression clustered by subject with and without a

constant term is shown in columns (1) and (2) in Table 7 respectively. The constant of regression (1) is insignificant. We therefore drop the constant term in regression (2). In regression (2), we cannot reject the null hypothesis that the coefficient of valuation is equal to one with a 95% confidence interval.

Table 7. Regression of exit bid on valuation

	(1)	(2)
Constant	2.050 (1.303)	-
Valuation	0.964* (0.173)	0.993* (0.005)
R-squared	0.950	0.998

* Significant at 95% confidence interval. Standard errors are shown in the parentheses. Sample size is 342.

5.2 Bidding behavior in treatment LAB

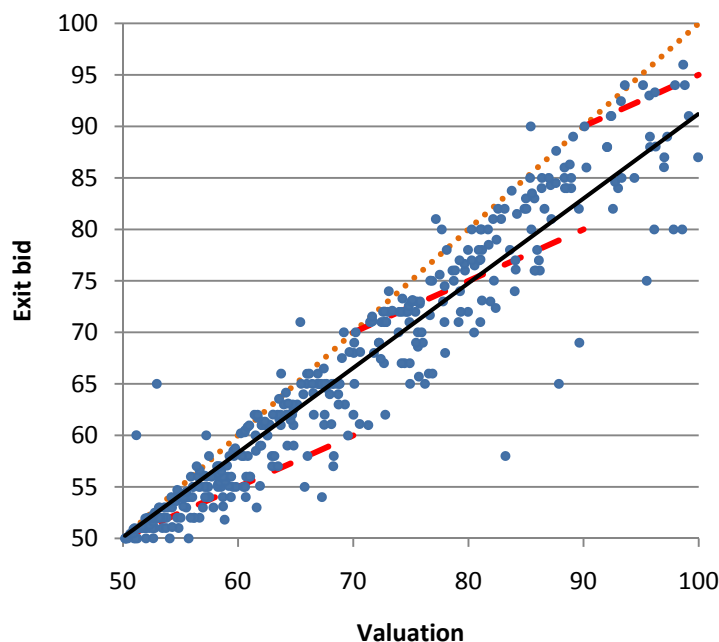
Table 8. Summary statistics of selected variables in treatment LAB

Variable	Observations	Mean	Std.Dev.
Valuation	640	75.54	14.71
Valuation if exit is submitted	386	69.23	13.29
Exit bid	386	65.82	12.00
Theoretical exit bid without fear of losing	386	64.82	12.77

Table 8 gives summary statistics of selected variables in treatment LAB. A total of 386 exit bids were submitted. Exit bids on average are lower than the valuations and slightly higher than the theoretical prediction. A scatter plot between valuations and exit bids is shown in Figure 4. The solid line is a linear estimation without an intercept, the dashed line is a theoretical exit bid function for profit maximizing bidders and the dotted line is the 45-degree line. Subjects shaded most of their exit bids by bidding below the 45-degree line but, as shown in Figure 2, the actual

seller's share of gain from trade per auction is substantially higher than the theoretical prediction without fear of losing. This is the case because subjects bid more aggressively than the theory predicted as shown in Figure 4 in which a large portion of actual exit bids lie above the theoretical exit bid function. We have proposed in Section 3 that such overbidding may stem from the fact that subjects anticipate regret of losing at an affordable price.

Figure 4. Plot between valuations and exit bids in LAB auction



Next, we will estimate the fear of losing coefficient from the bidding data by using the bidding function described in Proposition 5 (see Table 3). To estimate the fear of losing coefficient, we use maximum likelihood and non-linear estimations.

The likelihood function for observation i is $L_i(e_i|\alpha,\sigma^2) = N(e_i - b(\alpha,v_i),\sigma^2)$ where e_i is an observed exit bid and N is a normal distribution function with a variance of σ^2 . Maximizing the log likelihood function with respect to α and σ^2 , we find α is equal to 1.292 with a standard error of 0.289 and σ is equal to 4.424 with a standard error of 0.159. Therefore, we can reject the null hypothesis that the fear of losing coefficient is equal to zero with 95% confidence interval. Given this fear of losing coefficient, the expected revenue of LAB is equal to 68.83.

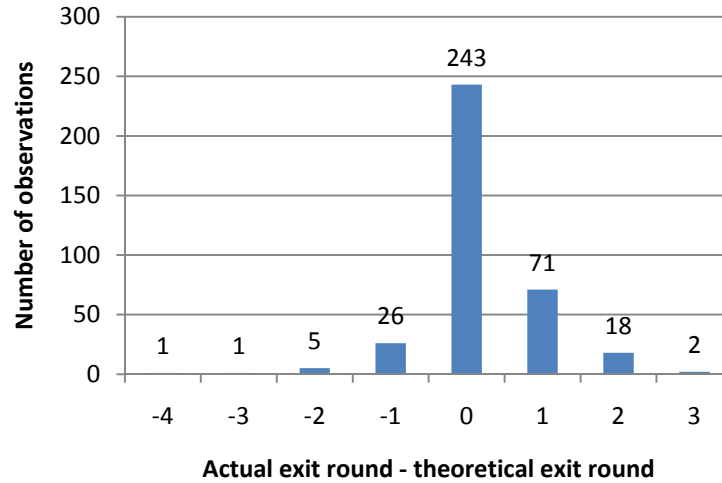
5.3 *Bidding behavior in treatment LABpw*

Table 9. Summary statistics of selected variables in treatment LABpw

Variable	Observations	Mean	Std.Dev.
Valuation	600	74.87	14.95
Valuation if exit	367	68.69	13.80
Exit price	367	62.48	11.97
Theoretical exit price without fear of losing	367	60.57	12.46

A total of 367 exit decisions are observed. The summary statistics are provided in Table 9. As predicted by the theory (in Propositions 3 and 6), the provisional losers stayed in provided her value was not yet reached (only 11 violations out of 600 auctions). Additionally, similar to treatment LAB, aggressive bidding behavior in treatment LABpw is observed, as indicated by the higher revenue than the theoretical prediction and a mean exit price that is higher than that of theoretical exit price. A histogram of differences between actual and theoretical exit round is provided in Figure 5. There are 91 instances of bidders staying in extra rounds versus 33 instances of bidders exiting early compared with the theory without fear of losing. In addition, among 233 active bidders at the end of the auction, 18 bidders overbid.

Figure 5. Histogram of differences between actual and theoretical exit round



To estimate fear of losing for treatment LABpw, we assigned the lowest coefficient α such that the theoretical bidding decisions match the actual observations from round one to the last round. We suspect that our estimated fear of losing coefficient is biased downward. The theoretical model cannot explain 75 out of 600 observations, 42 of which involve bidding above one's valuation and 11 of which are exiting before the valuation is reached when being a provisional loser. The remaining 22 observations involve exiting so early when being a provisional winner that even the theory with no fear of losing cannot justify the exit. By dropping the 75 irrational bidding observations, the mean of the fear of losing coefficient is 0.585 with standard error of 2.429. Given this coefficient, the expected revenue of LAB is equal to 66.59.

6 Conclusion

The pricing rule is of fundamental importance in practical auction design. Pricing impacts both the efficiency and the revenues of the auction. Although there is an immense literature on the pricing rule in static (sealed-bid) auctions—first-price vs. second-price in single unit auctions and pay-as-bid vs. uniform-price in multi-unit auctions—little is known about alternative pricing rules in dynamic auctions commonly used in practice. We showed how different pricing rules influence bidding behavior in discrete clock auctions in a simple setting.

Based on the standard theory in which bidders seek to maximize profits, the highest-rejected-bid (HRB) auction seems best. It maximizes revenues and is fully efficient in our unit-demand setting. In contrast, the lowest-accepted-bid (LAB) auction creates incentives for bid shading that complicate bidding and may reduce efficiency. Despite this theoretical result, LAB pricing is often used in practice. Behavioral economics provides an explanation for this choice. With the LAB auction, bid shading causes bidders to risk losing at profitable prices. Bidders who fear losing at profitable prices reduce their bid shading in order to lessen this risk. As a result, fear of losing may cause the LAB auction to have higher revenues than the HRB auction, even if efficiency is compromised. In our experiments, the LAB auction, both with and without exit bids, yielded higher revenues than the HRB auction. The HRB auction did better on efficiency grounds, but not significantly so when compared to LAB.

It is a robust finding of the experimental literature that in second-price sealed-bid auctions subjects bid more than their value although they bid truthfully in its dynamic counterpart, the English auction (Kagel et al. 1987, Kagel 1995, Kagel and Levin 1993). We experimentally found that bidders in HRB do not deviate from the straightforward bidding strategy. Therefore, it is confirmed that dynamic formats are easier for the bidders to play equilibrium strategies. It is important to note that HRB converges to the English auction (continuous clock) when the number of rounds approaches infinity. Moreover, when there is only one round, HRB is the same as the second-price auction. Having a discrete clock makes HRB more practical than the English auction, and given its success in eliciting the true valuations in our experiment, we recommend HRB over English and second-price auctions.

Despite the possibility of higher revenues from the LAB auction, we still recommend the HRB auction in situations like spectrum auctions, where efficiency should be the prominent objective. It is important to note that the revenue gains under LAB are offset by the inefficiencies. One source of inefficiency, which we have ignored so far, is bidder participation costs. Bidding strategy in the LAB auction is incredibly complex even in the simplest cases. In sharp contrast, bidding strategy in the HRB auction is simple in simple settings.

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Appendix: Bidder Instructions

Treatment HRB

Experiment Instructions

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions you can finish the experiment with a considerable amount of money, which will be paid to you in cash at the end. The experiment will last about 80 minutes.

The type of currency used in this experiment is Experimental Currency Units (ECU). Participants completing the session do not risk losing any money. At the end of the experiment all your earnings will be converted to US Dollars. The conversion rate is 10 ECU = 1 US Dollar. You will be paid in cash when you finish the experiment. The more ECU you earn, the more US Dollars you earn. If you participate in this experiment until the session is over, then you will be paid an additional 5 US Dollars.

Auction Rules and Calculation of Earnings

You will be participating in 20 auctions. At the beginning of the first auction, you will be randomly matched with another participant in this room; every auction, you will be randomly re-matched with a different participant. In each auction you will be bidding against the participant with whom you are matched.

In a given auction, there is a fictitious good that is sold and your aim is to bid in order to win this good. When the first auction starts, you will observe your valuation of the fictitious good. Your valuation is a number between 50 and 100 and it is randomly selected from the [50,100] interval with equal probability and rounded to the nearest cent. The other bidder participating in this auction also receives his or her independent valuation for the fictitious good and his or her valuation is also randomly selected from [50,100] interval. Each bidder will know only his or her own valuation.

The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. The computer will ask

you if you would like to stay in the auction when the price increases to 60. You either stay IN which indicates that you are willing to pay 60 for the good or you stay OUT. If you stay out for at a price of 60, you need to enter an exit bid which must be an amount between 50 and 60. The exit bid is the maximum amount that you are willing to pay for the good. For example, if you stay out and indicate that your exit bid is 54, then it means you would be willing to pay at most 54 for the good. The exit bid has to be an amount between the previous price level and the current price at which you are staying out. There are four possible things that can happen:

- You stay IN, and your opponent stays OUT with an exit bid of, say, 55: Then you win the fictitious good and pay 55, your opponent's exit bid.
- You stay OUT with exit bid of, say 57, and your opponent stays IN: Then your opponent wins the fictitious good and pays 57, your exit bid.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays the smaller of the exit bids.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 70.

If both you and your opponent stay in the auction for price level 60, then the computer will ask if you would like to stay in the auction when the price increases to 70. Again there are four possible things that can happen:

- You stay IN, and your opponent stays OUT with exit bid of, say, 65: Then you win the fictitious good and pay 65, your opponent's exit bid.
- You stay OUT with exit bid of, say 67, and your opponent stays IN: Then your opponent wins the fictitious good and pays 67, your exit bid.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays the smaller of the exit bids.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 80.

If both you and your opponent stay in the auction for price level 70, then the computer will ask if you would like to stay in the auction at a price of 80.

The same procedure will repeat for price levels of 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance and that bidder will pay 100.

Please note that the price level increases only if both bidders stay in the auction. If only one bidder stays in and the other one stays out with an exit bid, the bidder who is in wins the good and pays the exit bid of the other bidder. Otherwise, if both bidders stay out then the one with the higher exit bid wins the good and pays the exit bid of the other bidder. For example, when the price level is 70, if you indicate to stay OUT with exit bid of 63, and your opponent indicates to stay OUT with exit bid of 67, then your opponent wins the good (he or she has the higher exit bid) and pays 63 (which is the smaller exit bid). If you both submit the same exit bid while staying out then, the computer randomly assigns the good to one of the bidders, and the winner pays the exit bid of the other.

When one bidder wins the good, the auction is over. If you are the winner at a certain price, then you earn the difference between your valuation and the price. For example, let us say you have a valuation of 82.55 for the fictitious good in the current auction and you win the good at a price of 61. Then your earning from this round is

$$\text{Earning} = 82.55 - 61 = 21.55 \text{ ECU}$$

When the first auction is completed, the second auction will start. At the beginning of the second auction, you will be randomly matched with another participant in this room and play with that person in this round. First, the computer will show you your new valuation for the good for this auction. It is again a randomly selected number from [50,100] interval. Your opponent will also observe his or her own valuation for the good for this auction privately. The same auction rules as in the first auction will apply.

There are 20 auctions in total. The computer will sum up your earnings in ECU in all auctions and convert this amount to the US Dollars by dividing by 10. We will pay you this amount in cash at the end of the experiment in person.

In order to make sure that you understand the rules of the auction, we have a test period before the real session starts. In this test period, you will see some multiple choice questions about the auction rules. Please answer those questions to the best of your knowledge. You may look at the hard copy of the instructions while answering them. Once you answer all the questions correctly, one practice auction will be conducted for which no payment will be made. Then the experiment will start with 20 real auctions.

Please ask if you have any questions.

Questions for the test period (*asked to the subjects during the test period by the computer*)

- 1) The price level just moved from 60 to 70 and the computer asks you if you would like to stay in at a price of 70. You said that you are IN and your opponent said that he or she is OUT with an exit bid of 65. Then what would be the outcome of the auction?
- a) You would win the good and pay 60.
 - b) Your opponent would win the good and pay 70.
 - c) You would win the good and pay 65.
 - d) Nobody would win the good and the price would move to 80.

Answer: (c)

- 2) The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You decided to stay OUT for price level of 80. What are the possible exit bids you may enter?
- a) Any amount between 65 and 70.
 - b) Any amount between 70 and 80.
 - c) Any amount between 60 and 70.
 - d) Any amount between 80 and 90.

Answer: (b)

- 3) Let us say, your valuation of the good is 91 for the current round. The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You said that you are OUT with an exit bid of 77 and your opponent said that he or she is OUT with an exit bid of 74. Then what would be your and your opponent's earnings for this round?
- a) You would win the good and pay 77. You would earn 14 and your opponent would earn 0.
 - b) Your opponent would win the good and pay 74. You would earn 0 and you cannot know your opponent's earning without knowing his or her valuation.
 - c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.
 - d) You would win the good and pay 74. You would earn 17 and your opponent would earn 0.

Answer: (d)

Experiment Instructions

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions you can finish the experiment with a considerable amount of money, which will be paid to you in cash at the end. The experiment will last about 80 minutes.

The type of currency used in this experiment is Experimental Currency Units (ECU). Participants completing the session do not risk losing any money. At the end of the experiment all your earnings will be converted to US Dollars. The conversion rate is 10 ECU = 1 US Dollar. You will be paid in cash when you finish the experiment. The more ECU you earn, the more US Dollars you earn. If you participate in this experiment until the session is over, then you will be paid an additional 5 US Dollars.

Auction Rules and Calculation of Earnings

You will be participating in 20 auctions. At the beginning of the first auction, you will be randomly matched with another participant in this room; every auction, you will be randomly re-matched with a different participant. In each auction you will be bidding against the participant with whom you are matched.

In a given auction, there is a fictitious good that is sold and your aim is to bid in order to win this good. When the first auction starts, you will observe your valuation of the fictitious good. Your valuation is a number between 50 and 100 and it is randomly selected from the [50,100] interval with equal probability and rounded to the nearest cent. The other bidder participating in this auction also receives his or her independent valuation for the fictitious good and his or her valuation is also randomly selected from [50,100] interval. Each bidder will know only his or her own valuation.

The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. The computer will ask you if you would like to stay in the auction when the price increases to 60. You either stay IN

which indicates that you are willing to pay 60 for the good or you stay OUT. If you stay out at a price of 60, you need to enter an exit bid which must be an amount between 50 and 60. The exit bid is the maximum amount that you are willing to pay for the good. For example, if you stay out and indicate that your exit bid is 54, then it means you would be willing to pay at most 54 for the good. The exit bid has to be an amount between the previous price level and the current price at which you are staying out. There are four possible things that can happen:

- You stay IN, and your opponent stays OUT: Then you win the fictitious good and pay 60.
- You stay OUT, and your opponent stays IN: Then your opponent wins the fictitious good and pays 60.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays his or her exit bid.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 70.

If both you and your opponent stay in the auction for price level 60, then the computer will ask if you would like to stay in the auction when the price increases to 70. Again there are four possible things that can happen:

- You stay IN, and your opponent stays OUT: Then you win the fictitious good and pay 70.
- You stay OUT, and your opponent stays IN: Then your opponent wins the fictitious good and pays 70.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays his or her exit bid.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 80.

If both you and your opponent stay in the auction for price level 70, then the computer will ask if you would like to stay in the auction at a price of 80.

The same procedure will repeat for price levels of 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance and that bidder will pay 100.

Please note that the price level increases only if both bidders stay in the auction. If only one bidder stays in and the other one stays out, the bidder who is in wins the good and pays the price for which he or she indicated to stay in. Otherwise, if both bidders stay out then the one with the

higher exit bid wins the good and pays his or her exit bid. For example, when the price level is 70, if you indicate to stay OUT with exit bid of 63, and your opponent indicates to stay OUT with exit bid of 67, then your opponent wins the good (he or she has the higher exit bid) and pays 67 (his or her exit bid). If you both submit the same exit bid while staying out then, the computer randomly assigns the good to one of the bidders, and the winner pays his or her exit bid.

When one bidder wins the good, the auction is over. If you are the winner at a certain price, then you earn the difference between your valuation and the price. For example, let us say you have a valuation of 82.55 for the fictitious good in the current auction and you win the good at a price of 61. Then your earning from this round is

$$\text{Earning} = 82.55 - 61 = 21.55 \text{ ECU}$$

When the first auction is completed, the second auction will start. At the beginning of the second auction, you will be randomly matched with another participant in this room and play with that person in this round. First, the computer will show you your new valuation for the good for this auction. It is again a randomly selected number from [50,100] interval. Your opponent will also observe his or her own valuation for the good for this auction privately. The same auction rules as in the first auction will apply.

There are 20 auctions in total. The computer will sum up your earnings in ECU in all auctions and convert this amount to the US Dollars by dividing by 10. We will pay you this amount in cash at the end of the experiment in person.

In order to make sure that you understand the rules of the auction, we have a test period before the real session starts. In this test period, you will see some multiple choice questions about the auction rules. Please answer those questions to the best of your knowledge. You may look at the hard copy of the instructions while answering them. Once you answer all the questions correctly, one practice auction will be conducted for which no payment will be made. Then the experiment will start with 20 real auctions.

Please ask if you have any questions.

Questions for the test period (*asked to the subjects during the test period by the computer*)

- 1) The price level just moved from 60 to 70 and the computer asks you if you would like to stay in at a price of 70. You said that you are IN and your opponent said that he or she is OUT with an exit bid of 65. Then what would be the outcome of the auction?
- a) You would win the good and pay 70.
 - b) Your opponent would win the good and pay 70.
 - c) You would win the good and pay 65.
 - d) Nobody would win the good and the price would move to 80.

Answer: (a)

- 2) The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You decided to stay OUT for price level of 80. What are the possible exit bids you may enter?
- a) Any amount between 65 and 70.
 - b) Any amount between 70 and 80.
 - c) Any amount between 60 and 70.
 - d) Any amount between 80 and 90.

Answer: (b)

- 3) Let us say, your valuation of the good is 91 for the current round. The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You said that you are OUT with an exit bid of 77 and your opponent said that he or she is OUT with an exit bid of 74. Then what would be your and your opponent's earnings for this round?
- a) You would win the good and pay 77. You would earn 14 and your opponent would earn 0.
 - b) Your opponent would win the good and pay 74. You would earn 0 and you cannot know your opponent's earning without knowing his or her valuation.
 - c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.
 - d) You would win the good and pay 74. You would earn 17 and your opponent would earn 0.

Answer: (a)

Experiment Instructions

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions you can finish the experiment with a considerable amount of money, which will be paid to you in cash at the end. The experiment will last about 80 minutes.

The type of currency used in this experiment is Experimental Currency Units (ECU). Participants completing the session do not risk losing any money. At the end of the experiment all your earnings will be converted to US Dollars. The conversion rate is 10 ECU = 1 US Dollar. You will be paid in cash when you finish the experiment. The more ECU you earn, the more US Dollars you earn. If you participate in this experiment until the session is over, then you will be paid an additional 5 US Dollars.

Auction Rules and Calculation of Earnings

You will be participating in 20 auctions. At the beginning of the first auction, you will be randomly matched with another participant in this room; every auction, you will be randomly re-matched with a different participant. In each auction you will be bidding against the participant with whom you are matched.

In a given auction, there is a fictitious good that is sold and your aim is to bid in order to win this good. When the first auction starts, you will observe your valuation of the fictitious good. Your valuation is a number between 50 and 100 and it is randomly selected from the [50,100] interval with equal probability and rounded to the nearest cent. The other bidder participating in this auction also receives his or her independent valuation for the fictitious good and his or her valuation is also randomly selected from [50,100] interval. Each bidder will know only his or her own valuation.

The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. Then the computer will randomly determine one of you as a **provisional winner**. Each bidder has 50% chance of

being a provisional winner. Next, the computer will ask you if you would like to stay in the auction when the price increases to 60. There are four possible things that can happen:

- You stay IN, your opponent stays OUT: Then you win the fictitious good and pay 60.
- You stay OUT, your opponent stays IN: Then your opponent wins the fictitious good and pays 60.
- You stay OUT, your opponent stays OUT: Then the provisional winner wins the fictitious good and pays 50.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 70.

If both you and your opponent stay in the auction for price level 60, then the computer will randomly determine a new provisional winner and ask if you would like to stay in the auction when the price increases to 70. Again there are four possible things that can happen:

- You stay IN, your opponent stays OUT: Then you win the fictitious good and pay 70.
- You stay OUT, your opponent stays IN: Then your opponent wins the fictitious good and pays 70.
- You stay OUT, your opponent stays OUT: Then the provisional winner wins the fictitious good and pays 60.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 80.

If both you and your opponent stay in the auction for price level 70, then the computer will randomly determine a new provisional winner and ask if you would like to stay in the auction when price increases to 80.

The same procedure will repeat for price levels 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance, and the winner will pay 100.

Please note that the price level only increases if both bidders stay in the auction. Each time the price level increases a new provisional winner is randomly determined by the computer and each bidder has equal chance of being the provisional winner at that price level. The provisional winner wins the fictitious good at the previous price level if both bidders stay OUT at the current price level. Otherwise, if one bidder stays in and the other bidder stays out in the current round, then the bidder who is IN wins the good at the current price, regardless of the provisional winner designation.

When one bidder wins the good, the auction is over. If you are the winner at a given price, then you earn the difference between your valuation and the price. For example, let us say you have a valuation of 82.55 for the fictitious good in the current auction and you win the good at a price of 60. Then your earning from this round is

$$\text{Earning} = 82.55 - 60 = 22.55 \text{ ECU}$$

When the first auction is completed, the second auction will start. At the beginning of the second auction, you will be randomly matched with another participant in this room and play with that person in this auction. First, the computer will show you your new valuation for the good for this auction. It is again a randomly selected number from [50,100] interval. Your opponent will also observe his or her own valuation of the good for this auction privately. The same auction rules as in the first auction will apply.

There are 20 auctions in total. The computer will sum up your earnings in ECU in all auctions and convert this amount to US Dollars by dividing by 10. We will pay you this amount in cash at the end of the experiment in person.

In order to make sure that you understand the rules of the auction, we have a test period before the real session starts. In this test period, you will see some multiple choice questions about the auction rules. Please answer those questions to the best of your knowledge. You may look at the hard copy of the instructions while answering them. Once you answer all the questions correctly, one practice auction will be conducted for which no payment will be made. Then the experiment will start with 20 real auctions.

Please ask if you have any questions.

Questions for the test period (*asked to the subjects during the test period by the computer*)

- 1) The price level just moved from 60 to 70 and the computer determined **you** as the provisional winner for the current price. The computer asks you if you would like to stay in for price level 70. You said that you are IN and your opponent said that he or she is OUT. Then what would be the outcome of the auction?
- a) Your opponent would win the good and pay 70.
 - b) You would win the good and pay 70.
 - c) You would win the good and pay 60.
 - d) Nobody would win the good and the price would move to 80.

Answer: (b)

- 2) The price level just moved from 60 to 70 and the computer determined **your opponent** as the provisional winner for the current price. The computer asks you if you would like to stay in for price level 70. You said that you are OUT and your opponent said that he or she is OUT. Then what would be the outcome of the auction?
- a) You would win the good and pay 70.
 - b) Your opponent would win the good and pay 70.
 - c) You would win the good and pay 60.
 - d) Your opponent would win the good and pay 60.

Answer: (d)

- 3) Your valuation of the good is 91 for the current auction. The price level just moved from 70 to 80 and the computer determined **you** as the provisional winner for the current price. The computer asks you if you would like to stay in for price level 80. You said that you are OUT and your opponent said that he or she is OUT. Then what would be your and your opponent's earnings for this auction?
- a) You would win the good and pay 70. You would earn 21 and your opponent would earn 0.
 - b) Your opponent would win the good and pay 70. You would earn 0 and you cannot know your opponent's earning without knowing his or her valuation.
 - c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.
 - d) Your opponent would win the good and pay 80. You would earn 0 and your opponent would earn 15.

Answer: (a)