The Clock-Proxy Auction: A Practical Combinatorial Auction Design

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Introduction

- **Many related (divisible) goods**
  - **Airport slots (time, airport)**
  - **Spectrum (bandwidth, location)**
  - **Electricity (duration, location, strike price)**
  - **Financial securities (duration)**
  - **Emissions (duration, type)**
- **A practical combinatorial auction, as an alternative to the simultaneous ascending auction (SAA)**

Auction takeoff/landing slots at LaGuardia

In 2000, LaGuardia caused 25% of all US Delays

Application: Airport Slots

- Proposed design
  - 8 slots in each 15 minute period from 6:00am through 10:00pm, with one slot reserved for unscheduled flights
  - 20% auctioned each year with 5-year term
  - A slot provides the right to schedule an arrival within a given 15 minute period and a departure any time within 90 minutes after landing
  - Clock-proxy auction (now to be described)

Application: Spectrum Auction

- Trinidad and Tobago (23 June 2005)
  - Clock determines
    - Two license winners
    - Minimum price of bandwidth ($/block)
  - Proxy round determines size of licenses and specific band plan

Clock Auction

- Auctioneer names prices; bidders name only quantities
  - Price adjusted according to excess demand
  - Process repeated until market clears
- No exposure problem (package auction)

Proxy Auction

- A procedure for package bidding
  - Bidders input their values into "proxy agents"
  - Proxy agents iteratively submit package bids, selecting best profit opportunity according to the inputted values
  - Auctioneer selects provisionally-winning bids according to revenue maximization
  - Process continues until the proxy agents have no new bids to submit

Clock-Proxy Auction

- A clock auction, followed by a "final round" consisting of a proxy auction
  - Bidders directly submit bids in clock auction phase
  - When clock phase concludes, bidders have a single opportunity to input proxy values
  - Proxy phase concludes the auction

Clock-Proxy Auction

- All bids are kept "live" throughout auction (no bid withdrawals)
- Bids from clock phase are also treated as package bids in the proxy phase
- All bids are treated as mutually exclusive (XOR)
- Activity rules are maintained within clock phase and between clock and proxy phases

Advantages of Clock-Proxy Auction

- **Clock phase**
  - **Simple for bidders**
  - **Provides price discovery**
    - **Interdependent values**
    - **Economize on package evaluation costs**
- **Proxy phase**
  - **Efficient allocations**
  - **Competitive revenues**
  - **Reduces opportunities for collusion**

Clock Auction

Simultaneous Clock Auction

- Practical implementation of the fictitious "Walrasian auctioneer"
  - Auctioneer announces a price vector
  - Bidders respond by reporting quantity vectors
  - Price is adjusted according to excess demand
  - Process is repeated until the market clears

Simultaneous Clock Auction

- Strengths
  - Simple for bidders
  - Provides highly-usable price discovery
  - Yields similar outcome as SAA, but faster and fewer collusive opportunities
  - A package auction without complexity
- Weaknesses
  - Limits prices to being linear
  - Therefore should not yield efficient outcomes

Recent Clock Auctions

- EDF generation capacity (virtual power plants)
  - 16 quarterly auctions (Sep 2001 – present)
- Electrabel generation (virtual power plants)
  - 7 quarterly auctions (Dec 2003 – present)
- Ruhrgas gas release program
  - 3 annual auctions (2003 – present)
- Trinidad and Tobago spectrum auction
  - 1 auction (June 2005)
- Federal Aviation Administration airport slot auction
  - 1 demonstration auction (Feb 2005)
- UK emissions trading scheme
  - World's first greenhouse gas auction (Mar 2002)
- GDF and Total gas release program
  - 2 auctions (Oct 2004)

Recent Clock Auctions

- New Jersey basic generation service
  - 5 annual auctions (2002 – present)
- Texas electricity capacity
  - 16 quarterly auctions (Sep 2001 – present)
- Austrian gas release program
  - 3 annual auctions (2003 – present)
- Nuon generation capacity
  - 1 auction (September 2004)

EDF Generation Capacity Auction

Typical EDF Auction

- Number of products
  - Two to four groups (baseload, peakload, etc.)
  - 20 products (various durations)
- Number of bidders
  - 30 bidders
  - 15 winners
- Duration
  - Eight to ten rounds (*one day*)
- €300 million in value transacted in auction

Electrabel VPP Capacity Auction

Electrabel

MDI
market design inc.

Typical Electrabel Auction

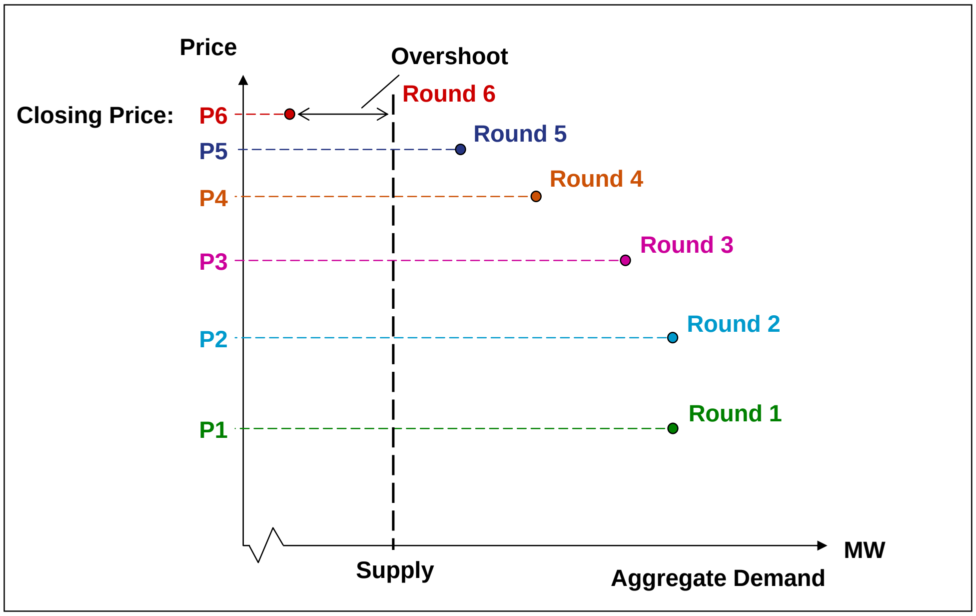
- Number of products
  - Two groups (baseload, peakload)
  - 20 products (various durations and start dates)
- Number of bidders
  - 14 bidders
  - 7 winners
- Duration
  - Seven rounds (*one day*)
- €100 million in value transacted in auction

Issues in Implementing Clock Auctions

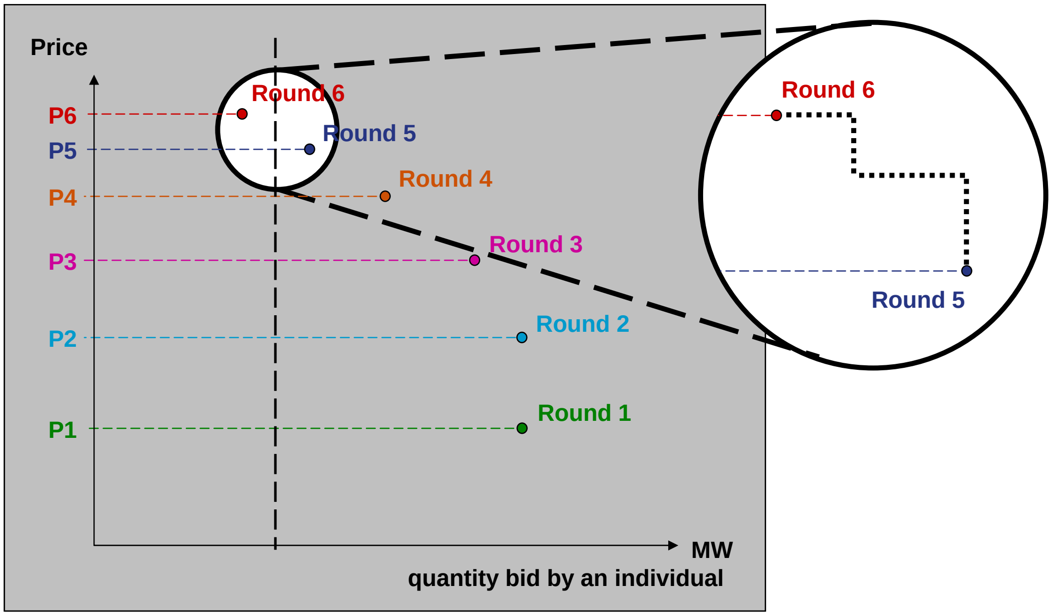
Issue 1: Discrete bidding rounds are helpful for maintaining legally-binding bids, but they can yield slow auctions or "overshoot"

- **SOLUTION: Intra-round bids: If the (end) price of Round 3 is €19,000 and the (end) price of Round 4 is €19,500 for baseload, and if the (end) price of Round 3 is €10,300 and the (end) price of Round 4 is €10,600 for peakload, then bidders in Round 4 submit demand curves for all price pairs from (€19,000 , €10,300) to (€19,500 , €10,600).**

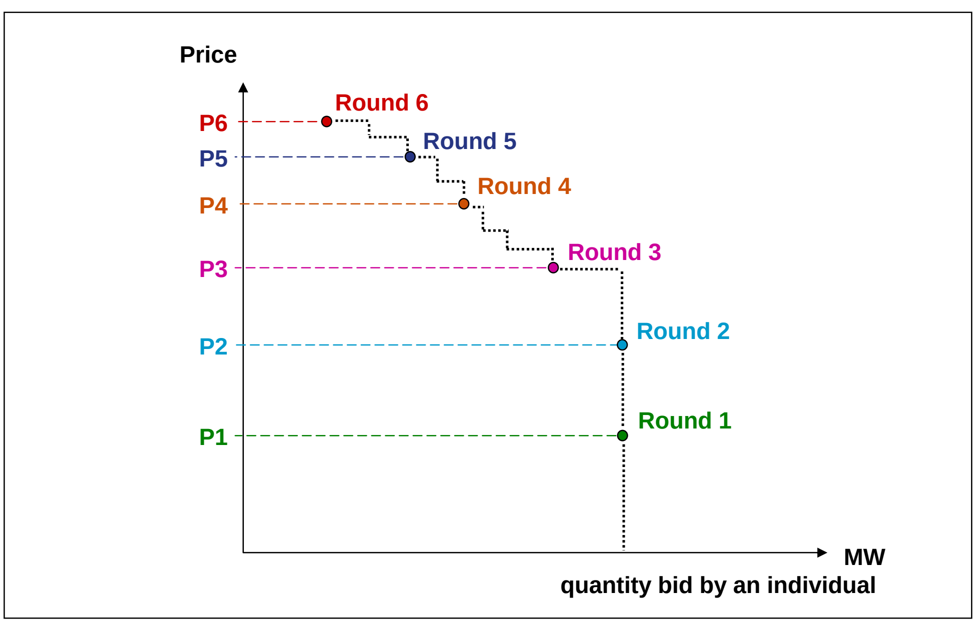
1 Product – Dealing with Discreteness



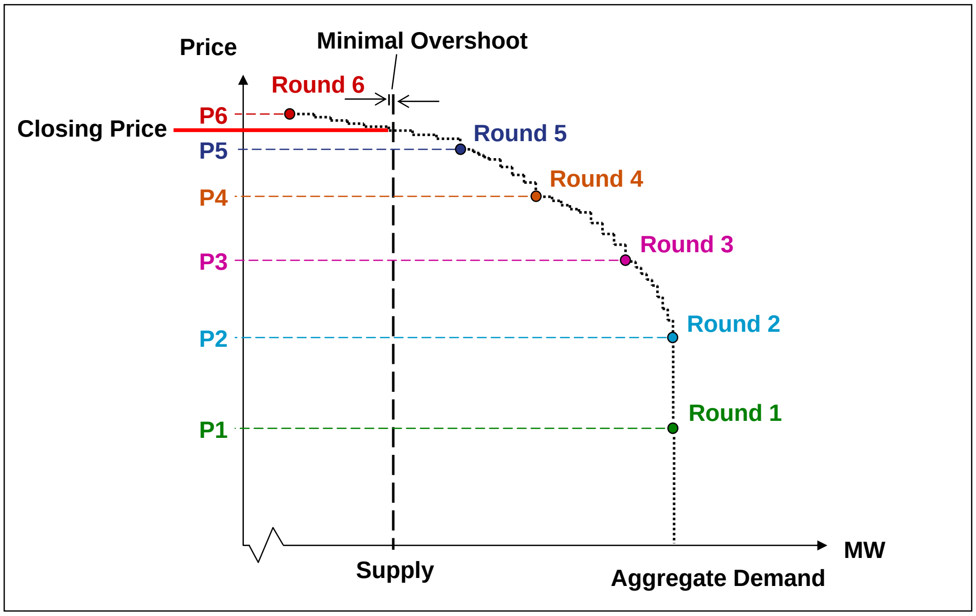
1 Product introducing intra-round bidding



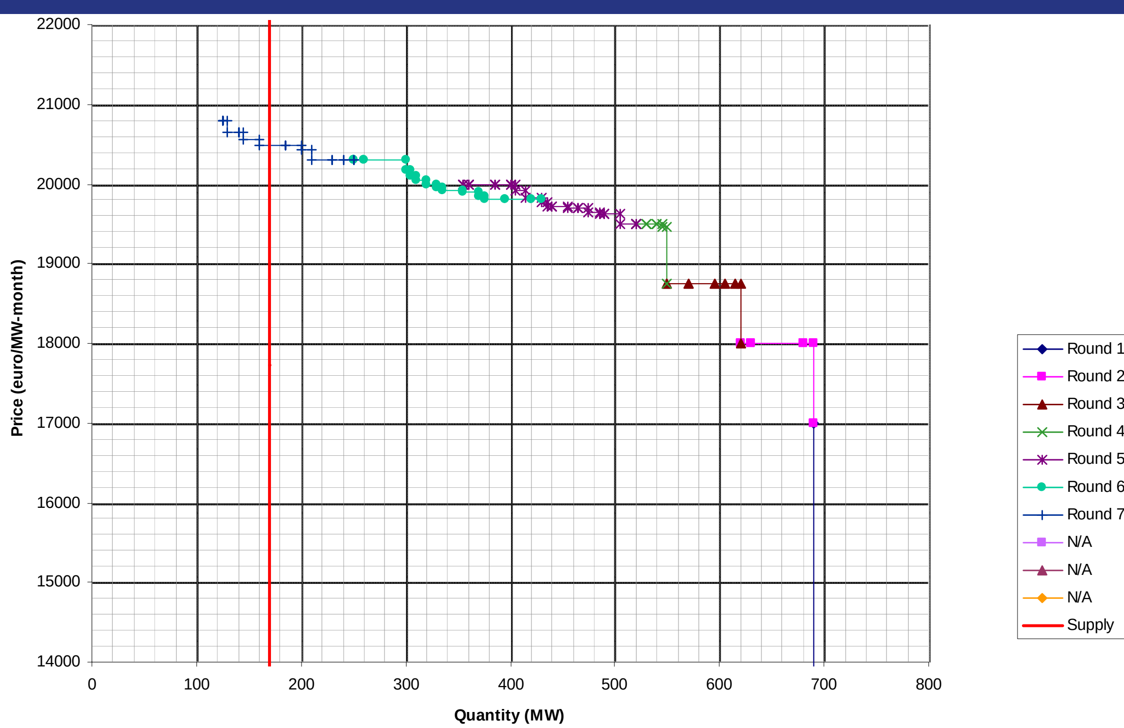
1 product – Individual bids with intra-round bidding



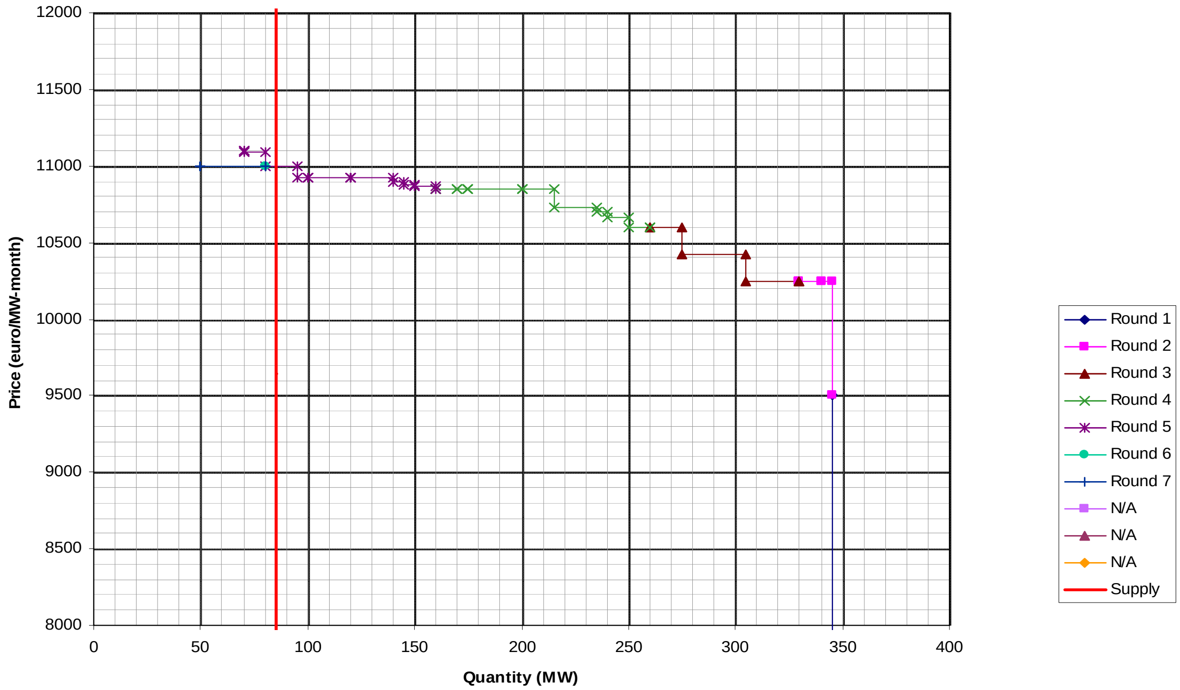
1 product – Aggregate demand with intra-round bidding



Sample (redacted) data 1



Sample (redacted) data 2



Issues in Implementing Clock Auctions

Issue 2: Treatment of bids which would make aggregate demand < supply

- **Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce its demand**
  - **Naive approach: Prevent the reduction**
- **Example: For a particular item, demand > supply, but demand < supply at next increment**
  - **Naive approach: Ration the bidders**

Issues in Implementing Clock Auctions

Issue 2: Treatment of bids which would make aggregate demand < supply

- **Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce its demand**
  - Difficulty: Creates an exposure problem
- **Example: For a particular item, demand > supply, but demand < supply at next increment**
  - Difficulty: Creates an exposure problem

Issues in Implementing Clock Auctions

Issue 2: Treatment of bids which would make aggregate demand < supply

- **Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce its demand**
  - Our approach: Allow the reduction
- **Example: For a particular item, demand > supply, but demand < supply at next increment**
  - Our approach: No rationing

Issues in Implementing Clock Auctions

Issue 2: Treatment of bids which would make aggregate demand < supply

- Bids in clock phase are treated as package bids
- Thus, our clock auctions are, in fact, combinatorial auctions
- Advantage: No exposure problem
- Disadvantage: Potential significant undersell (*But not a problem in the clock-proxy auction, since clock phase followed by a final proxy round*)

Issues in Implementing Clock Auctions

Issue 3: Activity rules

- Prevent a bidder from hiding as a "snake in the grass" to conceal its true interests
- Standard approaches:
  - No activity rule (laboratory experiments)
  - Monotonicity in quantities (SAA and clock auctions in practice)

Issues in Implementing Clock Auctions

Issue 3: Activity rules

- Revealed-preference activity rule (advocated here)
- Compare times *s* and *t* (*s* < *t*),
  Prices: p^s, p^t Demands: x^s, x^t
  - At time *s*, x^s is better than x^t: $v(x^s) - p^s \cdot x^s \geq v(x^t) - p^s \cdot x^t$
  - At time *t*, x^t is better than x^s: $v(x^t) - p^t \cdot x^t \geq v(x^s) - p^t \cdot x^s$
  - Adding inequalities yields the RP activity rule:

$$(RP) \qquad (p^t - p^s) \cdot (x^t - x^s) \leq 0 .$$

Issues in Implementing Clock Auctions

Issue 3: Activity rules

- Revealed-preference activity rule (advocated here)
- Bid placed at time *t* must satisfy (RP) with respect to its prior bids at all prior times *s* (*s* < *t*):

$$(RP) \qquad (p^t - p^s) \cdot (x^t - x^s) \leq 0 .$$

- One can also apply a “relaxed” RP in proxy phase (with respect to bids in the clock phase)

Proxy Auction

Package Bidding

- Package bidding often motivated by complements
- Even without complements, package bidding may improve outcome by eliminating “demand reduction”
  - In SAA, bidders may have strong incentives to reduce demands in order to end auction at low prices

Ascending Proxy Auction

- **Each bidder reports its values (and constraints) to a "proxy agent", in a sealed-bid round**
- **The proxy agents bid in an auction in "virtual time"**
- **The proxy agent's rule: submit the allowable bid that, if accepted, would maximize the bidder's payoff (evaluated according to its reported values)**
- **The virtual auction ends after a round with no new bids by the proxy agents**

Outcomes in the Core

- **The coalitional form game is (*L*,*w*), where…**
- **L denotes the set of players.**
  - **the seller is *l* = 0**
  - **the other players are the bidders**
- ***w*(*S*) denotes the value of coalition *S*:**
  - **If S excludes the seller, let *w*(*S*)=0**
  - **If S includes the seller, let**

$$w(S) = \max_{x \in X} \sum_{l \in S} v_l(x_l)$$

- **The *Core*(*L*,*w*) is the set of all profit allocations that are *feasible* for the coalition of the whole and *cannot be blocked* by any coalition *S***

Outcomes in the Core

<u>Theorem</u>: The payoff vector resulting from the proxy auction is in the core relative to the reported preferences.

Interpretations:

- Core outcome assures competitive revenues for seller
- Core outcome assures allocative efficiency (ascending proxy auction is not subject to inefficient demand reduction)

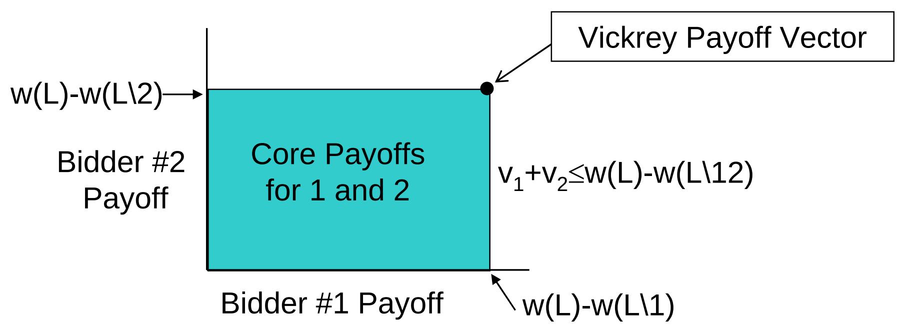
Outcomes in the Core

<u>Theorem</u>: If π is a bidder-Pareto-optimal point in *Core*(*L*,*w*), then there exists a full information Nash equilibrium of the proxy auction with associated payoff vector π.

These equilibria may be obtained using strategies of the form: bid your true value minus a nonnegative constant on every package

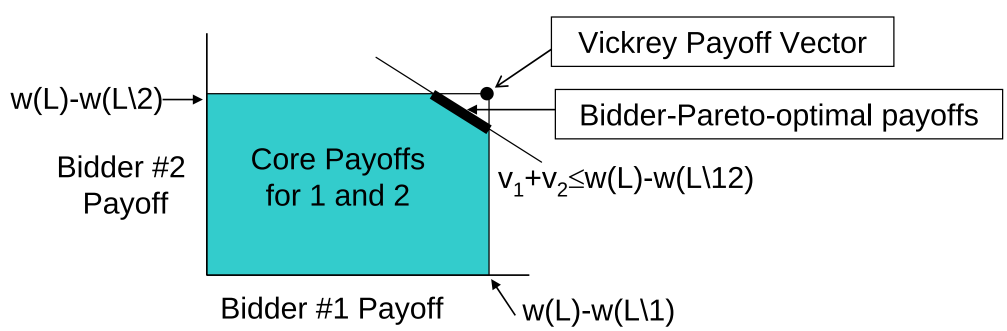
Case of Substitutes

- If goods are substitutes, then Vickrey payoff profile is unique bidder-Pareto-optimal point in core
- Outcome of the ascending proxy auction coincides with outcome of the Vickrey auction



Case of Non-Substitutes

- If goods are not substitutes, then Vickrey payoff profile is not in core
- Ascending proxy auction yields a different outcome from the Vickrey auction (one with higher revenues)



Proxy Auction Avoids Vickrey Problems

- In Vickrey auction:
  - Adding a bidder can reduce revenues
  - Using a shill bidder can be profitable
  - Losing bidders can profitably collude
- Proxy auction avoids these problems

Clock-Proxy Auction

Clock-Proxy Auction

- A simultaneous clock auction is conducted, with a revealed-preference activity rule imposed on bidders, until (approximate) clearing is attained
- A proxy auction is conducted as a "final round"
  - Bids submitted by proxy agents are restricted to satisfy a relaxed revealed-preference activity rule based on competitive conditions
  - Bids from clock phase are also treated as "live" package bids in proxy phase
  - All package bids (clock and proxy) are treated as mutually exclusive, and auctioneer selects as provisionally-winning the bids that maximize revenues

Relaxed Revealed Preference Activity Rule

- Let s be a time in clock phase and t a time in proxy phase
- Package S is bid on at time s and T is bid on at time t
- $P^s(S)$ and $P^s(T)$ package prices of S and T at time s
- $P^t(S)$ and $P^t(T)$ package prices of S and T at time t
- At every time t in the proxy phase, the bidder can bid on the package T only if (RRP) is satisfied for every package S bid at time s in the clock phase
- (RRP) $\alpha[P^t(S) - P^s(S)] \geq P^t(T) - P^s(T)$
- $\alpha > 1$ is parameter (closer to 1 if more competitive environment)
- For $\alpha = 1$, price of S increased more than price of T; otherwise S would be more profitable than T.
- Alternatively, state RRP as a constraint on valuations reported to proxy:

$$v(T) - P^s(T) \leq \alpha\left(v(S) - P^s(S)\right)$$

Why Not Use the Proxy Auction Only?

- Clock auction phase yields price discovery
- Feedback of linear prices is extremely useful to bidders
- Clock phase makes bidding in the proxy phase vastly simpler
  - Focus decision on what is relevant
  - See what you don't need to consider
  - See what looks like good possibilities

Why Not Use the Clock Auction Only?

- Proxy auction ends with core outcome
  - Efficient allocation
  - Competitive revenues
- No demand reduction
- Collusion is limited
  - Relaxed activity rule means allocation still up for grabs in proxy phase

Advantages of the Clock over the SAA

- Clock auction is a fast and simple process (compared to the simultaneous ascending auction)
  - Only provide information relevant for price and quantity discovery (excess demand)
  - Takes advantage of substitutes (one clock for substitute licenses)
  - Example:
    - proposed 90 MHz of 3G spectrum in 5 blocks: 30, 20, 20, 10, 10
    - clock alternative: 9 or 18 equivalent blocks per region
  - Fewer rounds
    - Get increment increase for all items, rather than having to cycle through over many rounds
    - “Intra-round bids” allow larger increments, but still permit expression of demands along line segment from start-of-round price to end-of-round price

Advantages of the Clock over the SAA

- Clock auction limits collusion (compared to the simultaneous ascending auction)
  - Signaling how to split up the licenses greatly limited
    - No retaliation (since no bidder-specific information)
    - No stopping when obvious split is reached (since no bidder specific information)
  - Fewer rounds to coordinate on a split

Advantages of the Clock Phase

- No exposure problem (unlike SAA)
  - As long as at least one price increases, bidder can drop quantity on other items
  - Bidder can safely bid for synergistic gains
  - Bid is binding only as full package
- Limited threshold problem (unlike ascending package auction)
  - Clocks controlled by auctioneer: no jump bids; large bidder cannot get ahead
  - Linear pricing: small bidders just need to meet price on single item

Clock-Proxy Auction

- Combines advantages of
  - Clock auction
  - Proxy auction
- Excellent price discovery in clock phase simplifies bidder decision problem
- Proxy phase enables bidders to fine-tune allocation based on good price information

Advantages of Clock-Proxy Auction

- **Clock**
  - Take linear prices as far as they will go
  - Simplicity and flexibility for bidders and auctioneer
  - Expand substitution possibilities
  - Minimize scope for collusion
  - No exposure problem; no threshold problem
- **Proxy**
  - Core outcome
    - Efficiency
    - Substantial seller revenues