

# Setting the Upset Price in British Columbia Timber Auctions

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## SUMMARY

An important element of timber auctions is the upset—the minimum acceptable price, often called the reserve price in other auction environments. The upset has three main purposes: (1) to guarantee substantial revenue in auctions where competition is weak but the upset is met, (2) to limit the incentive for—and the impact of—collusive bidding, and (3) to provide useful information to bidders. We analyze the determination of the upset in British Columbia timber auctions. Setting the upset too high results in unsold stands and produces an upward bias in price if the competitive auctions are used to determine stumpage rates for non-auctioned timber. Setting the upset too low will reduce auction revenue and can create downward bias when the auction prices are used to calculate timber prices for non-auctioned stands. It is therefore important to set the upset at or near the optimal level. We present the theory of upset pricing and then apply that theory to the data available from historical timber auction sales in the BC Interior from 1999 to 2000. We find that an upset of about 70 percent (a rollback of 30 percent) maximizes auction revenues if the Ministry values timber at about 52 to 56 percent of its appraised value. This upset strikes the right balance between enhanced revenues and unsold timber stands. Given its importance, this upset calibration should be refined as additional data becomes available to assure that the upset is not set too high or too low.

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## 1 INTRODUCTION

The British Columbia Ministry of Forests asked us to analyze the choice of the upset in British Columbia timber auctions. The upset, commonly called the reserve price in other auction environments,<sup>2</sup> has three important functions: (1) it assures that auction revenues will be substantial even if competition is weak provided the upset is met, (2) it reduces the incentive for, and impact of, collusive bidding, and (3) it provides useful pricing information to potential bidders.

Determining a suitable upset is important in any auction. But in the British Columbia timber auctions, the upset's importance is magnified, because an upset that is too high can upwardly bias the timber prices paid for non-auctioned stands. In particular, the prices for non-auctioned timber are estimated using the auction prices from successful auctions. An aggressively high upset will lead to a significant percentage of unsuccessful auctions—auctions without any bidders willing to meet or exceed the upset. Those auctions are eliminated when estimating the prices for non-auctioned timber. However, those observations occur when the market value is lower than expected, and therefore the average price at successful auctions is greater than the average market value of all auctioned timber. Such pricing creates an upward bias when successful auction sales are used to price non-auctioned timber. Similarly, setting an upset that is too low can downwardly bias the price estimates for non-auctioned timber. Theory predicts that bidders will bid less aggressively when the upset is too low. Furthermore, an inadequate upset creates greater scope for market manipulation and collusive bidding. Given the importance of the upset in the B.C. timber auctions, we recommend that the Ministry of Forests recalibrate the upset periodically. After the first year of operation of the program, the upset should be reviewed. It also can be done when the pricing model is reestimated, which likely would occur every few years. At times of structural change, it may be necessary to temporarily adjust the upset on an emergency basis. If many stands are unsold the Ministry should consider lowering the upset.

We begin by presenting the theory of upset pricing. We then apply the theory to the historical timber auctions in the British Columbia Interior from 1999 to 2000. Our analysis suggests that an upset of 70 percent (a rollback of 30 percent) maximizes auction revenues if the Ministry values timber at about 52 to 56 percent of its appraised value.

## 2 THE THEORY OF UPSET PRICING

Consider a seller auctioning off a single item (a stand of timber) to  $n$  risk-neutral bidders, indexed by  $i = 1, \dots, n$ . Each bidder,  $i$ , has a privately known value for the item  $v_i$ . Neither the seller, nor the other bidders, know  $i$ 's value; they know only that it is drawn from the probability distribution  $F$  with positive density  $f$  on support  $[0, h]$ , independently from the other values  $v_j, j \neq i$ . This model is called the symmetric independent private values model, since each bidder looks the same to the seller ex ante, and each bidder has a private value for the good, which is drawn independently from other bidders' values. The seller's value is denoted  $v_0$ .

The seller receives a set of bids, and then chooses whether to accept them. In any equilibrium to the bidding game, the bidding strategies are strictly increasing. Thus, a 1-1 mapping between bids and values exists. It is easier to describe the seller's problem in terms of the bidders' values. So, we think of the seller as receiving a vector of bids  $\mathbf{b}$ , inferring what the corresponding values are  $\mathbf{v}$ , and then determining if one of the bidders receives the item, or rather if the seller keeps it. That is, the seller chooses  $a_i(\mathbf{v}) \in \{0, 1\}$ , where  $a_i(\mathbf{v}) = 1$  indicates that bidder  $i$  is receiving the item. If  $a_i(\mathbf{v}) = 0$  for all  $i$ , the seller keeps the item.

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2. In this paper we use the term “upset” to indicate the minimum bid price that the Ministry will accept for a given timber stand. We use the term “reservation value” to indicate the Ministry's estimate of a stand's value if that stand were either resold at a later auction or harvested by the Ministry itself.

Using this formulation, one can show that the seller's profits equal

$$(1) \quad E_{v_1, \dots, v_n} \left[ \sum_{i=1}^n a_i(\mathbf{v}) \left( v_i - v_0 - \frac{1 - F(v_i)}{f(v_i)} \right) \right]$$

The derivation of this result is beyond the scope of this paper, but it is standard in the literature; see, Myerson (1981) or Bulow and Roberts (1989). Hence, to maximize profits, the seller allocates the item to the bidder that has the highest marginal revenue:

$$(2) \quad MR(v_i) = v_i - v_0 - \frac{1 - F(v_i)}{f(v_i)}.$$

Marginal revenue is less than the gains from trade (the gap between the value of the item to the buyer and the seller). The reason is that the bidder is able to capture some profits as a result of the bidder's private information. For example, in a first-price sealed-bid auction, the bidder shades its bid below its true value in order to maximize its profits. Bid shading allows the bidder to extract a portion of the gains from trade.

By setting an upset price above  $v_0$ , the seller eliminates bid shading by a bidder with a value equal to the upset. By implication, the seller also reduces the bid shading of bidders with values above the upset, since these bidders are competing to win the item from other bidders that are bidding more aggressively as a result of the upset. The incentive to bid high increases with a reduction in the likelihood that a low bid wins the auction. By refusing to sell the item even at bids above the seller's value, the seller makes it less tempting to bid low, and bid shading is reduced.

It is standard to assume that  $MR(v_i)$  is increasing in  $v_i$ . This is called the regular case, and holds for many common probability distributions (uniform, exponential, etc.). In the regular case, the seller maximizes profits by selling to the bidder with the highest marginal revenue, provided the marginal revenue is non-negative. Hence, the upset price is set at  $r$  such that  $MR(r) = 0$ ; in other words,  $r$  is the value that solves

$$(3) \quad r - v_0 = \frac{1 - F(r)}{f(r)}.$$

Clearly, it will not be worthwhile for any bidder with values below  $r$  to bid at or above the upset, whereas all bidders with values greater than  $r$  gain some surplus by bidding above the upset. A bidder with value  $r$  simply places a bid equal to  $r$ .

In the regular case, since  $MR(v_i)$  is increasing in  $v_i$ , if the seller finds it optimal to award the item to anyone, the seller awards it to the bidder with the highest value  $v_{\max} = \max \{v_1, \dots, v_n\}$ . Symmetry implies this result. The seller assigns the good efficiently—to maximize the gains from trade—subject to the constraint that the seller never sell at a price below  $r$ .

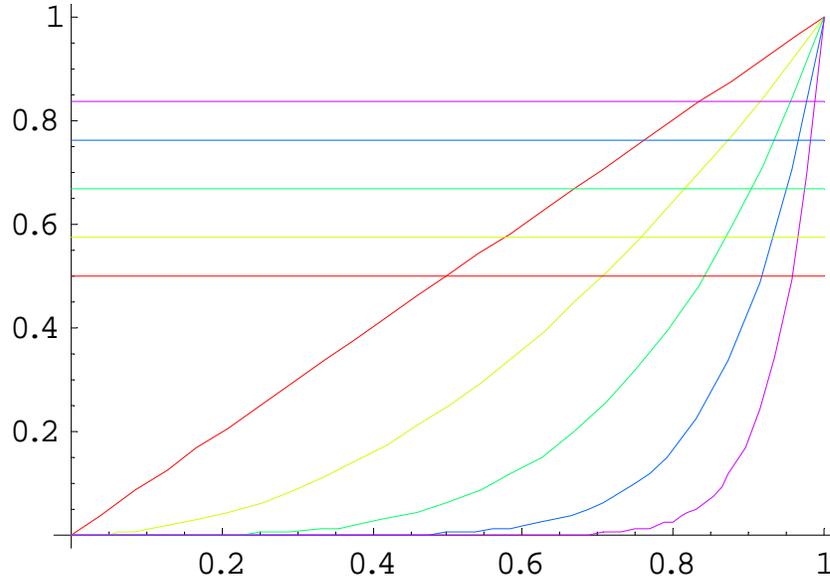
In the benchmark model, the optimal upset price does not depend on the number of bidders. This result is a good first-approximation even when we relax one or more of the assumptions of the benchmark model. The purpose of the upset is to guarantee substantial revenues even when competition is weak. Indeed, the optimal upset is the same as the optimal take-it-or-leave-it offer that the seller would make to a single buyer with a value  $v_i$  drawn from the distribution  $F$ .

To get a sense of how the optimal upset varies with the distribution  $F$ , consider the example where  $v_0 = 0$  and  $v_i$  is distributed according to the distribution  $F(v_i) = v_i^\alpha$ , on  $[0, 1]$ . For  $\alpha = 1$ , values are uniformly

distributed,  $MR(v_i) = 2v_i - 1$ , and the optimal upset is  $r = \frac{1}{2}$ . Notice that although the upset does not change with  $n$ , the probability of trade,  $F(r)^n = \frac{1}{2^n}$ , is very much dependent on the number of bidders.<sup>3</sup>

As the seller gets more confident that the expected value is near one, the upset increases. However, the probability of trade also increases, since the no-trade option becomes less attractive to the seller as the expected gains from trade increase. Figure 1 and Table 1, below, illustrate these two points. Figure 1 displays the optimal upset and the distribution function  $F$  for  $\alpha = 1, 2, 4, 8,$  and  $16$ . Table 1 displays the expected highest value, the optimal upset, and the probability of trade with  $n = 4$ , for  $\alpha = 1, 2, 4, 8,$  and  $16$ .

**FIGURE 1. VALUE DISTRIBUTION FUNCTIONS AND THE CORRESPONDING OPTIMAL UPSET**



The five horizontal lines in Figure 1 represent the optimal upset prices that correspond to the five upward sloping lines, which are the value distribution functions,  $F(v)$ . The red upward sloping line is the distribution function when  $\alpha = 1$ , and therefore  $F(v) = v$ . The upward sloping purple line corresponds to  $F(v) = v^{16}$ . The horizontal purple line at an approximate value of .84 represents the optimal upset for the value distribution  $F(v) = v^{16}$ . Because the difference between the distribution function evaluated at any two points along the x-axis represents the probability that a bidder draws a value between those two x-axis points, the movement from the red upward sloping line to the purple upward sloping line indicates that bidders are more likely to have higher values. Put differently, when the distribution function is  $F(v) = v$ , the probability of a bidder having a value between .95 and .9 is  $F(.95) - F(.9) = .05$ . If the distribution function is  $F(v) = v^{16}$ , however, the similar calculation becomes  $(.95)^{16} - (.9)^{16} = .255$ . There is a 5 percent chance of a bidder having value between .9 and .95 if the distribution function is  $F(v) = v$ , but there is a 25.5 percent chance that a bidder has a value between .9 and .95 if the distribution is  $F(v) = v^{16}$ . The upset increases as the distribution puts more weight on higher values. Now, consider Table 1 below, which displays the expected highest value, the optimal upset, and the probability of trade with  $n = 4$ , for  $\alpha = 1, 2, 4, 8,$  and  $16$ .

3. The probability of trade is the  $\text{Prob}\{\text{at least one bidder has value above } r\} = 1 - \text{Prob}\{\text{all bidders have value below } r\}$ . Thus, the probability of trade is as follows:  $\text{Prob}\{\text{trade} \mid r = \frac{1}{2}\} = 1 - F(r)^n = 1 - \frac{1}{2^n}$ .

**TABLE 1. OPTIMAL AUCTION CHARACTERISTICS WITH 4 BIDDERS**

$\alpha$	Expected High Value	Optimal Upset	Probability of Trade
1	0.800	0.500	0.938
2	0.889	0.58	0.988
4	0.941	0.67	0.998
8	0.970	0.76	0.999
16	0.985	0.84	0.999

The expected high value<sup>4</sup> increases with  $\alpha$ , because higher values are more likely when  $\alpha$  increases. The probability of trade also increases with  $\alpha$ .

In the model above, the optimal upset does not depend on the number of bidders. This result requires that the bidders are not colluding. However, if the bidders are colluding to keep the auction price low, then the optimal upset increases with the number of bidders. In the extreme case, the  $n$  bidders are colluding perfectly, with all bids at or just above the upset, provided one of the bidders has a value above the upset. This situation is equivalent to a seller selling to a single bidder with valuation drawn from the distribution  $F^n$ . The optimal upset in this case solves

$$(4) \quad r - v_0 = \frac{1 - F(r)^n}{nF(r)^{n-1} f(r)}$$

Table 2 shows the expected high value, the optimal upset when  $v_0 = 0$ , and the probability of trade for the uniform distribution on  $[0,1]$  by the number of bidders from 1 to 16.

**TABLE 2. OPTIMAL AUCTION WITH COLLUDING BIDDERS**

Number of Bidders, $n$	Expected High Value	Optimal Upset	Probability of Trade
1	0.500	0.50	0.500
2	0.667	0.577	0.667
4	0.800	0.669	0.800
8	0.889	0.760	0.889
16	0.941	0.838	0.941

When bidders collude, the optimal upset increases with the number of bidders. Because the expected high value increases with the number of bidders, the gains from trade will also rise with the number of bidders. However, the cartel will capture all incremental gains from trade if the upset remains constant when the cartel adds new members. To capture a share of the incremental gains, the seller must increase the upset price.<sup>5</sup>

Since we believe that British Columbia has established the necessary laws and enforcement to discourage collusion, we focus on the non-collusive case in the next section. The underlying theory and a discussion of the estimation technique is presented in Appendix B. Appendix A presents an example showing how an optimal upset is determined from sample data.

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4. The expected high value is nothing more than the expectation of the *highest* order statistic.

5. The reader may notice that the optimal upset prices in Table 1 and 2 are the same. This is simply a mathematical coincidence of this particular example. The fact that the expected high value and the probability of trade are equal in Table 2 is also an artifact of this particular example.

### 3 APPLYING THE THEORY TO ESTIMATE THE OPTIMAL UPSET IN BRITISH COLUMBIA

The analysis above indicates that the optimal upset price depends critically on the shape of the distribution of bidder valuations. A growing literature in applied econometrics is devoted to estimating such value distributions. This research uses auction theory and auction data (bids, the number of bidders, etc.) to estimate the underlying distribution from which bidders draw their values. Once the value distribution is estimated, the optimal upset price can be determined using the theory from the benchmark model. Below, we discuss the variables a researcher would require to perform this estimation. We then outline the estimation, and present our results. Our estimation is based on auctions conducted under the Small Business Forest Enterprises Program (SBFEP) in the BC Interior from 1999 to 2000.

The BC timber auctions are first-price sealed-bid auctions.<sup>6</sup> The Ministry currently records the winning bid, rather than all the bids. The Ministry also records the number of bidders at auction.

In our analysis, we assume that the bidders are symmetric in the sense that each bidder's value is drawn from the same probability distribution. Because the dataset represents loggers only, this assumption is reasonable. In the future, however, both loggers and tenure holders may bid, in which case the Ministry should consider applying an asymmetric bidder assumption (covered in Athey and Haile (2002) for the case when only the winning bid is observed). We also assume that conditional on the appraised value of the tract, bidders' valuations are independent of one another. This assumption is commonly made in the literature, and is reasonable if the appraised value incorporates most of the important characteristics of the tract that affect harvesting costs and the value of the timber.

The non-parametric estimation for a first price auction with symmetric bidders and independent private values is studied in Guerre, Perrigne, and Vuong (GPV) (1995) and GPV (2000). GPV (1995) provide distinct estimation procedures for two alternative scenarios: (1) an explicit upset price already exists in the market and (2) only the transaction price is observed. GPV (2000) shows the asymptotic properties of the non-parametric estimator when all the bids are observed. Here, we combine the two procedures in the 1995 paper to correct for the fact that the SBFEP auctions already have an upset and only the winning bid is observed.<sup>7</sup>

The nonparametric procedure is multi-step. First, we estimate the number of potential bidders. The number of potential bidders is equal to the number of bidders that would have placed a bid in the absence of an upset price. However, because some bidders have values below the upset, only a subset of the potential bidders actually places a bid in the auction. Following GPV (1995), as a proxy for the number of potential bidders, we use the maximum of the observed number of bidders in a given group (i.e. timber areas, which are pooled districts with similar characteristics). Because the bid function depends on the number of potential bidders, the estimation procedure is performed separately for auctions with different numbers of potential bidders.

We assume that the values, call them  $v_i$  are generated *multiplicatively* through the appraisal,  $\alpha$ , as follows:

$$v_i = \alpha \zeta_i^{\xi},$$

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6. See Paarsch (1997) for an analysis of BC's earlier oral auctions.

7. In the future, we recommend that the Ministry should collect data about all bids, rather than just the winning bid, because the estimation procedure will be far simpler and more powerful. However, to remove any incentives bidders might have to place "low-ball" bids that have little chance of success, in the hope of manipulating future reserve price calculations, it may be prudent to use only the top two or three in the estimation. Athey and Haile (2002) show how such data can be incorporated.

where  $\xi_i$  is an *iid* error term. Hence, we take the log of both sides to get the standard additive model:

$$\log(v_i) = \log(\alpha) + \varepsilon_i,$$

where  $\varepsilon_i = \log(\xi_i)$ . As we show below, a non-stochastic component in the values will carry through directly to the bids. Therefore, if one were to subtract the log of the appraisal from the log of the bids, one would have then constructed *iid* bids. Thus, our non-parametric analysis is performed on the *percent variation of the bids away from the appraised stand value*. Similarly, our “value estimates” are really estimates of the *variation of the corresponding values away from the appraised stand value*. To simplify language below, we will refer to these values as “bid disturbances” and “value disturbances.” A bid disturbance of -.25 indicates that the winning bid was 25 percent below the appraised value. An estimated value disturbance of .1 translates to an estimated value that was 10 percent above the appraisal. We now proceed with an overview of the estimation method.

The estimation method proceeds by using a kernel to estimate the distribution of winning bid disturbances. The theory of equilibrium bidding specifies that each bidder must respond optimally to the anticipated distribution of opponent bid disturbances. In an independent private values auction, the distribution of disturbances conveys enough information for the researcher to calculate the distribution of bids that any given bidder expects to face at auction. This distribution differs from the distribution of winning bids, since a bidder’s own bids enter into the latter distribution. Then, one can determine for each bid disturbance, the value disturbance and corresponding value that would make its bid optimal.

Formally, suppose that there is no upset price, so that the number of potential bidders is known and fixed at  $N$ , and suppose that the distribution of winning bids is  $G^{(1)}(b)$ . Then, the distribution of a typical bidder’s bids is  $G(b) = (G^{(1)}(b))^{\frac{1}{N}}$ . Anticipating this distribution of the maximum rival bid, a bidder with value  $v$  solves:

$$\max_b (v - b)(G(b))^{N-1},$$

since the bidder will win with bid  $b$  when all  $N-1$  opponents bid less than  $b$ . Taking the first-order condition, the bid  $b$  corresponding to value  $v$  must solve

$$(v - b)(N - 1)(G(b))^{N-2} g(b) - (G(b))^{N-1} = 0,$$

or, rearranging,

$$(5) \quad v = b + \frac{G(b)}{(N - 1)g(b)}.$$

We use this relationship to infer what the distribution of values must be. In particular, we use a kernel regression to estimate the right-hand side of (5), using the observed bids. Then, for every observed bid  $b$ , we substitute that  $b$  into our estimate of the right-hand side of (5), and infer what the value  $v$  must have been to satisfy the equation. Given this constructed dataset of inferred or pseudo values, we can then use a kernel density estimator to estimate the probability density of the value distribution. In turn, we integrate the density to obtain an estimate of the distribution of values.

In Appendix B, we outline the methodology in more detail, and show how it extends to the case where there is an upset price. We also discuss how we subtract the non-stochastic portion of the bids (assumed to be the appraised stand value) and run the estimation on the resulting bid disturbances.

### 3.1 Results from nonparametric estimation for the Interior

To account for regional characteristics in the auctions, we run the nonparametric estimation by geographic areas. The areas are Forest Districts whose bid data we pooled together because common bidders tend to compete within the combined area. We list these areas, and the associated Forest Districts in Table 3.<sup>8</sup>

**TABLE 3. ESTIMATION AREAS AND ASSOCIATED INTERIOR DISTRICTS**

<b>Bidding Area</b>	<b>Districts Comprising the Area</b>	<b>Observations</b>
1	Fort Nelson, Fort St. John, Dawson Creek	59
2	MacKenzie, Prince George	30
3	Prince George, Robson Valley, Quesnel, Ft. St. James, Vanderhoof	96
4	Bulkley-Cassier, Kispiox, Kalum	61
5	Quesnel, Horsefly, Williams Lake, 100 Mile House, Chilcotin, Vanderhoof	78
6	Lillooet, Kamloops, Clearwater, Merritt	53
7	Salmon Arm, Vernon, Penticton, Boundary	27
8	Columbia, Invermere, Cranbrook	19
9	Arrow, Boudary, Kootenay Lake	33
10	Lakes, Maurice, Vanderhoof	41

Certain bidding areas contain the same Forest District. For example, Prince George District is included in both Area 2 and Area 3. The overlap between areas will not affect our estimation within a single area, because we perform the estimation separately for each area.

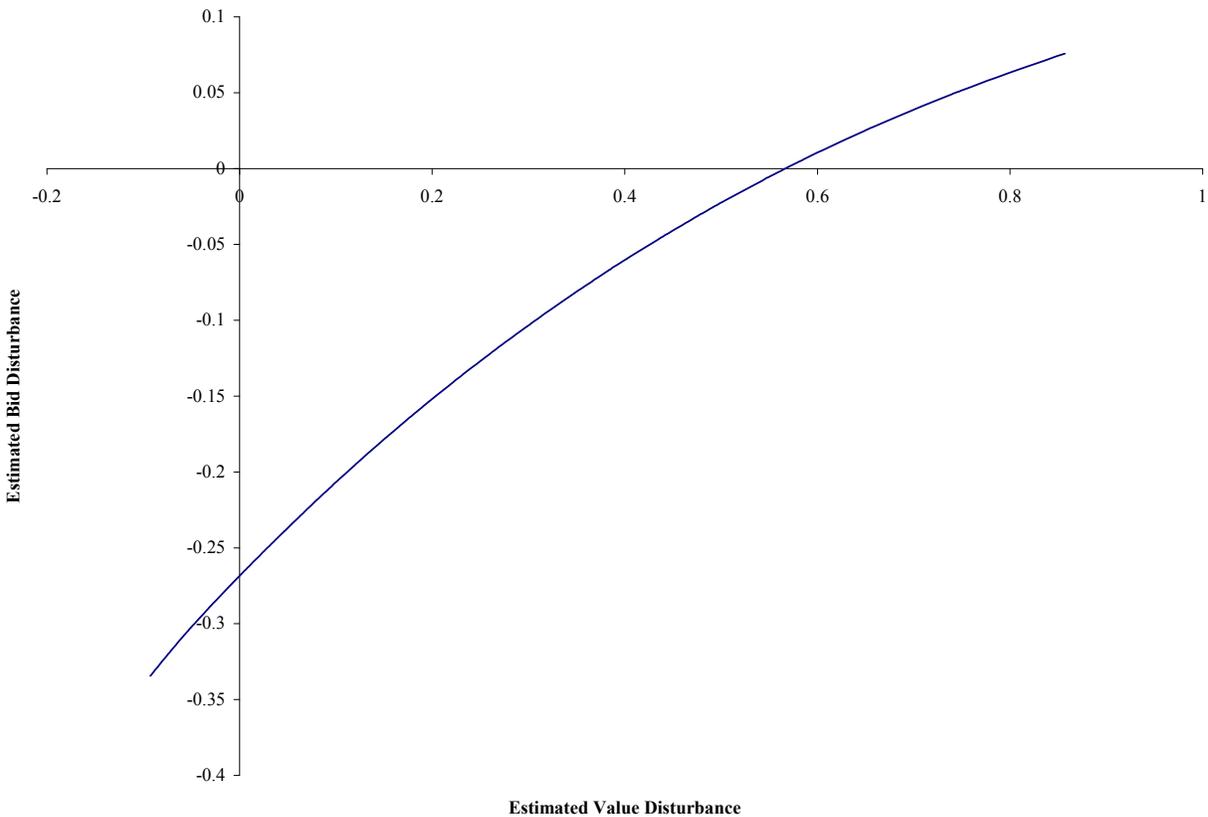
We first estimate the potential number of bidders in the area as the maximum number of bidders that bid for any single auction in that area. We then assume a probability of any bidder coming to the auction at an upset rate of 70 percent. Ideally, this probability should be calculated from the data, but given current size limitations this proves difficult. Thus, we proceed by varying this probability and re-calculating our results, noting if substantial changes occur.

After a kernel density estimation of the observed bids, we then retrieve the pseudo values—that is, the estimates of the underlying values. These steps are also done individually, by area. As an example, the bid function that we estimated for Area 3 (Prince George, Robson Valley, Quesnel, Ft. St. James, and Vanderhoof districts) is shown in Figure 2.

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8. Gurrere, Perrigne, and Vuong (2000), show that sample sizes of 200 generate highly accurate results. Our estimation contains sample sizes much less than 200. In the future, more data should be collected to increase the sample size, and improve the accuracy of the estimates.

**FIGURE 2. ESTIMATED BID FUNCTION FOR PRINCE GEORGE, ROBSON VALLEY, QUESNEL, FT. ST. JAMES, AND VANDERHOOF DISTRICTS**



In Figure 2, we see that the bid disturbances—the variation of the log bids away from the log of the appraised stand values—in Area 3 varied from between 33 percent below the appraisal and 8 percent above the appraisal.<sup>9</sup> Given this variation, we estimate that the underlying values varied from between 9 percent below the appraisal and 86 percent above the appraisal.<sup>10</sup>

An important specification test for the estimation procedure is to verify that the inverse bid function is monotonic. Only a monotonic bid function is consistent with theory. (Note that we plot the inverse bid function with values on the x-axis and bids on the y-axis, so that it can be interpreted as a bid function; a non-monotonicity in the inverse bid function corresponds to a “backward bend” in the curve.) Above we see that the estimated bid function is increasing, as required. We can therefore proceed to the next step, more confident in our results.

Given the estimated inverse bid functions, such as those displayed in Figure 2 above, we then compute the pseudo-value for each bid. The pseudo-values, and corresponding percent variations away

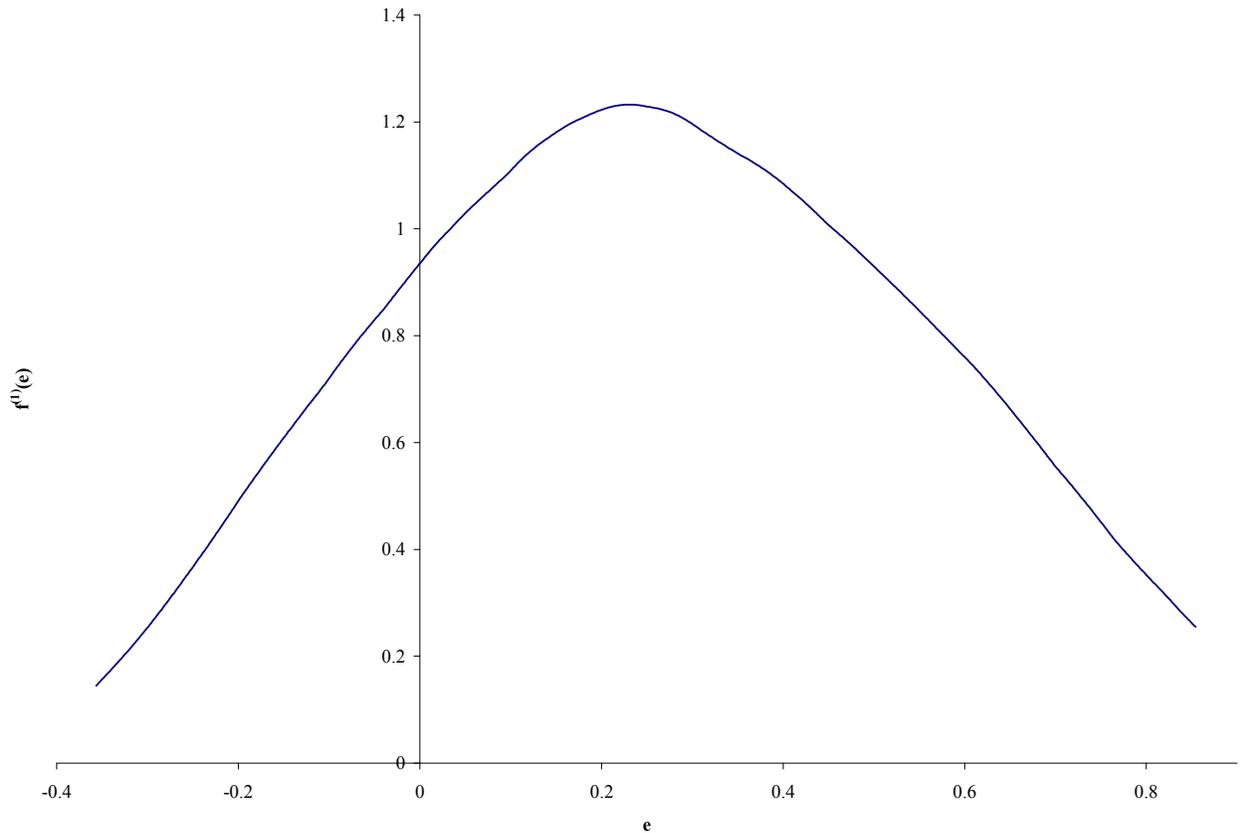
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9. Because we express the disturbances in log form, the percentages we describe differ from the percentages the ministry uses. In particular, a log bid disturbance that is 33 percent below the appraisal represents a bid that exceeds the 30 percent rollback used for the upset. The upset is determined as the product of 0.7 and the appraised value. The log of 0.7 is -.36. Thus, a log bid disturbance that is 33 percentage points below the appraisal corresponds to a bid that exceeds the upset.

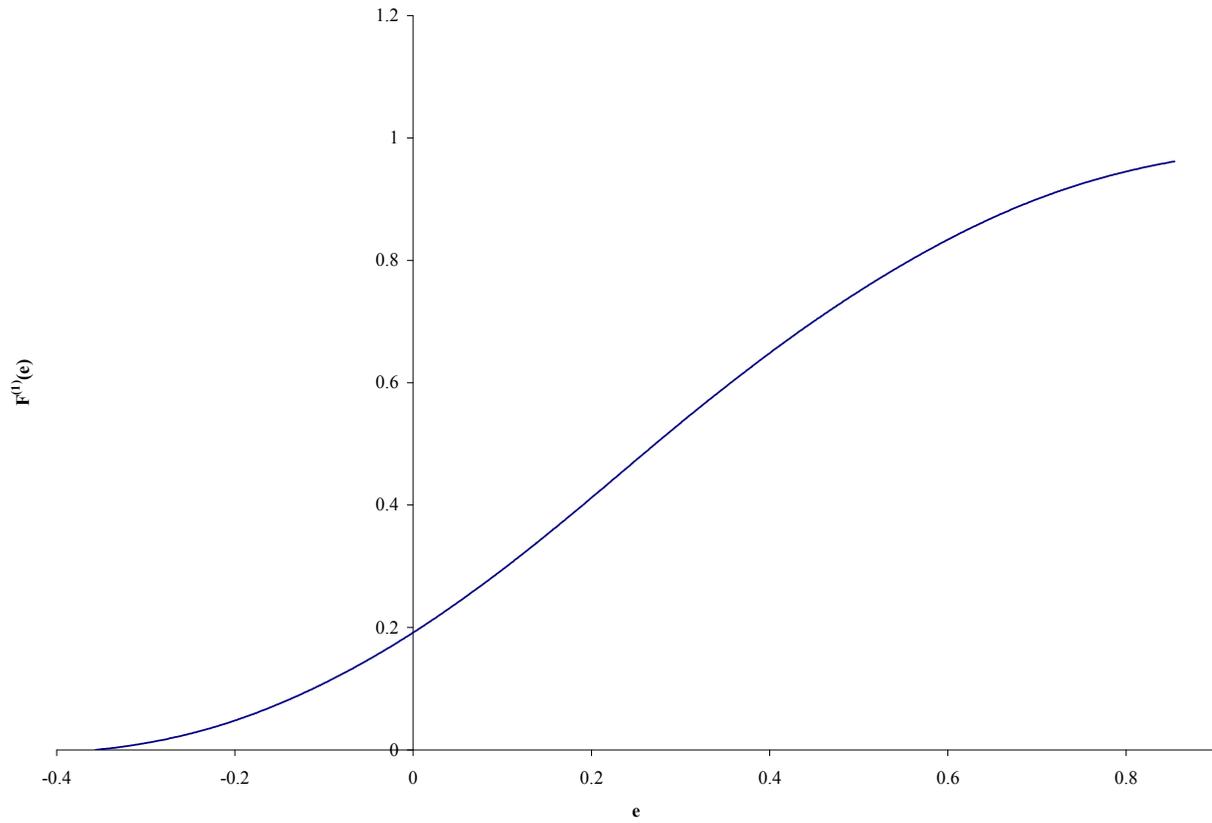
10. Our estimate of the number of potential bidders in area 3 is nine. On average, over four bidders submitted bids in this area.

from the appraisals, can then be used to estimate the density and distribution functions of those pseudo-values. A kernel density estimator and numerical integration are used for this procedure. Because we only observe the winning bids, we plot the density and distribution of the highest ordered value (among all of the bidders). We then transform these densities and distributions so that they correspond to the value distribution of a typical bidder, rather than the distribution for the highest-value bidder. Finally, the resulting distributions can be used to determine the optimal upset. Figures 3, 4, and 5 contain graphs of these estimates. Figure 3 contains the estimated density of the highest pseudo value in Area 3, while Figure 4 contains the corresponding distribution estimate. Then, in Figure 5, we present the estimated distribution of the underlying value disturbances.

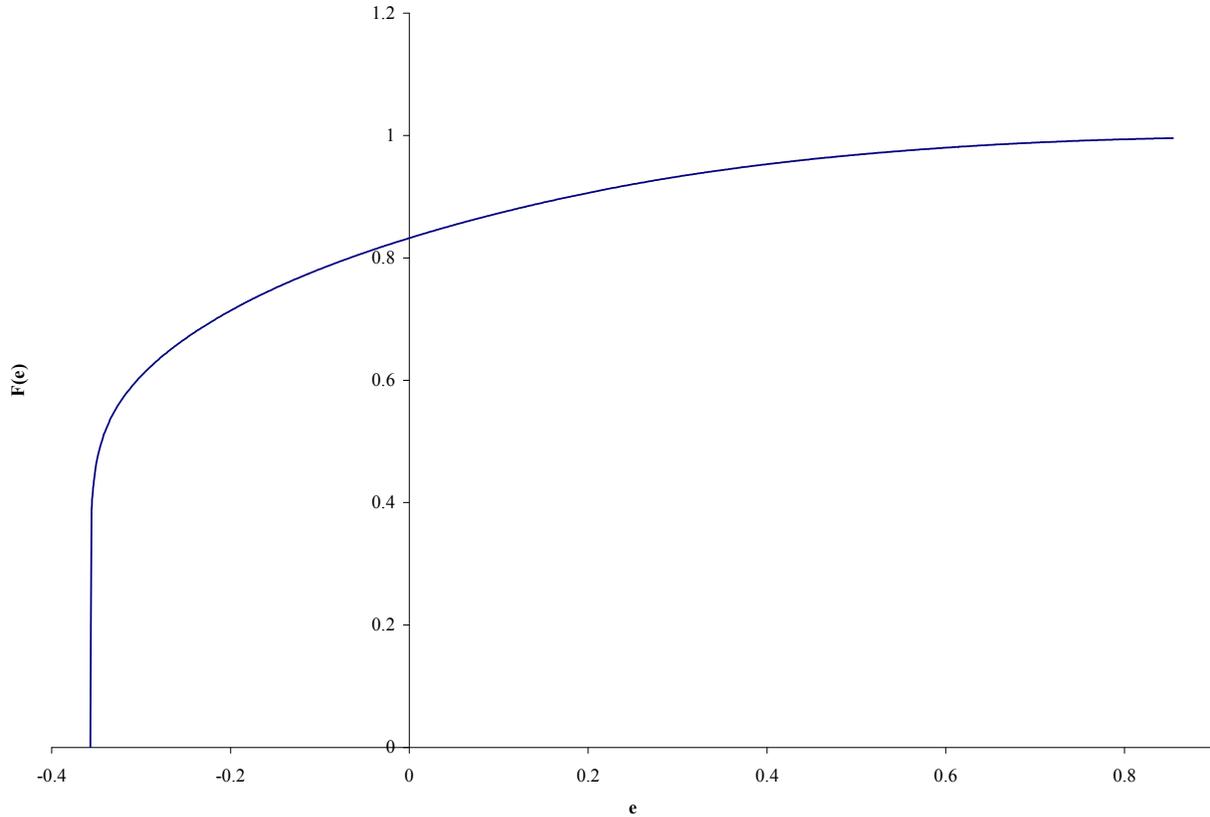
**FIGURE 3. ESTIMATED DENSITY OF VALUE DISTURBANCES FOR PRINCE GEORGE, ROBSON VALLEY, QUESNEL, FT. ST. JAMES, AND VANDERHOOF**



**FIGURE 4. ESTIMATED DISTRIBUTION OF THE HIGHEST VALUE DISTURBANCES FOR PRINCE GEORGE, ROBSON VALLEY, QUESNEL, FT. ST. JAMES, AND VANDERHOOF**



**FIGURE 5. ESTIMATED DISTRIBUTION OF THE VALUE DISTURBANCES FOR PRINCE GEORGE, ROBSON VALLEY, QUESNEL, FT. ST. JAMES, AND VANDERHOOF**



In Figures 3 and 4, we see that the kernel process estimates a smooth density and distribution function for Area 3. However, lumps in the underlying density do occur in some areas. The estimation is more challenging in those areas. For example, the bid density in Area 5 was bimodal even at high bandwidths, making it difficult to extract information for this Area.

### 3.2 Using the estimates to determine the optimality of the upset rule

Recall from above that the marginal revenue of the bidders is

$$MR(v_i) = v_i - v_0 - \frac{1 - F(v_i)}{f(v_i)},$$

when values are expressed in levels. However, we chose to estimate the logs of the bids rather than the bids themselves, and therefore must account for this transformation in the marginal revenue equation. Let  $H$  be the probability distribution for the log of the bidder values. Then,

$$MR(\log(v_i)) = \log(v_i) - \log(v_0) - \frac{1 - H(\log(v_i))}{h(\log(v_i))}.$$

Recall that  $\varepsilon$  is the stochastic component of the log values, so that  $\varepsilon_i = \log(v_i/\alpha)$ . Let  $\Phi$  be the distribution function of  $\varepsilon$  and let  $\phi$  be the density. Then, we have

$$MR(\log(v_i)) = \log(v_i) - \log(v_0) - \frac{1 - \Phi(\log(v_i/\alpha))}{\phi(\log(v_i/\alpha))}.$$

We now take  $\rho$  to be the upset *rate* (for the Ministry, this is .7), so that the minimum bid accepted will be  $\alpha \cdot \rho$ . Then, we must set an optimal upset that solves

$$(6) \log(\rho) - \frac{1 - \Phi(\log(\rho))}{\phi(\log(\rho))} = \log(v_0) - \log(\alpha)$$

The optimal upset depends critically on the Ministry's reservation value  $v_0$  for the stand, expressed as a percentage of appraised value. Since it is impossible to estimate the optimal upset rate  $\rho$  without knowing the reservation value  $v_0$ , we instead fix  $\rho$  and estimate the reservation value that would yield an optimal upset rate of  $\rho$ . Also, since the actual upset rate was 70 percent, which truncated the bid disturbances that we observe at 70 percent, we can only perform the estimation for values of  $\rho$  greater than 70 percent. We therefore consider an upset rate of 79 percent<sup>11</sup> and ask the question, "What Ministry reservation value,  $v_0$ , would make such an upset rate optimal?" Using 20 percent as the probability of drawing a value disturbance less than the reserve rate, we find that an upset rate of 79 percent coincides with a reservation stand value that is between 55.3 percent and 56.9 percent of the appraised stand value. Put differently, the Ministry would need to value the timber for its own purposes at between 55.3 percent and 56.9 percent of the appraised value for an upset rate of 79 percent to be optimal. These estimates are precise; the bootstrap standard errors are small, and the bootstrap confidence intervals indicate that the Ministry's true reservation value is between 52 and 62 percent of the appraised value with 95 percent probability.<sup>12</sup> We present these results in Table 4 for the areas where we are able to estimate the underlying value distribution with a comfortable degree of precision.

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11. 79 percent is the smallest upset for which the left-hand side of equation (6) allows us to estimate the implicit reservation value for Areas 1, 2, 3, 4, 6, and 10.

12. For a discussion of the methodology used to obtain the bootstrap standard errors and confidence intervals, see Section 2 of the Appendix A.

**TABLE 4. INFERRED MINISTRY RESERVATION VALUES, ASSUMING AN OPTIMAL UPSET OF 79 PERCENT**

Area	Reservation Value as Percent of Appraisal, Assuming Optimal 79% Upset	Bootstrap Standard Error	Bootstrap 95% Confidence Interval (Percentile Method)	Average Appraised Value (\$)	Inferred Ministry Reserve for Optimal 79% Upset (\$)
1	56.9	1.7	(53.4, 60.1)	35.57	20.24
2	55.9	1.9	(52.8, 61.6)	57.87	32.35
3	56.5	0.8	(54.8, 58.1)	55.25	31.22
4	55.3	1.5	(52.8, 58.5)	40.16	22.21
6	56.2	1.8	(53.0, 59.9)	49.60	27.88
10	56.6	1.6	(53.8, 60.4)	51.02	28.88

Results in Table 4 indicate that the Ministry’s upset rule signifies a higher *opportunity cost* of auctioning timber than the simple costs of reforestation, and land management. The Ministry is internalizing its alternative of letting the timber at auction. In particular, it could cut the timber itself and sell the end product, or it could hold the timber for sale at a later auction. The Ministry would then make more money through this policy than it would by accepting the statutory minimum for its timber. To better gauge the optimal upset, one must better understand the Ministry’s true reservation value relative to its private-sector appraisal. However, we caution that for low Ministry reservation values, we will be unable to calculate the optimal upset rule, because data in the relative portion of the value distribution will be unavailable. That is, we cannot infer the shape of the value distribution in regions of the distribution that are censored by the upset price.

Extrapolating from Table 4, we run a linear regression on our optimal upset results in the range .74 to .99, and then infer back to obtain the value of  $v_0$  that implies an optimal upset of 70 percent.<sup>13</sup> In particular, we apply a quadratic specification to the data, and estimate that equation using least squares. We obtain the regression coefficients, and then predict what the reservation rate would be at an optimal upset of 70 percent if had we been able to observe those data. Table 5 contains these results.

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13. The regression range is different for each Area, but each range is 20 percentage points wide. For example, we could have determined the reservation rate for an optimal upset of .75 in Area 1. Thus, we use the range .75 to .95 for the linear regression, and predict the optimal upset at 70 percent. Likewise, we could only determine the reservation rate for an optimal upset of 79 percent in Area 4 because of data restrictions. The regression sample in Area 4 therefore ranges from upset rates between .79 and .99.

**TABLE 5. LEAST SQUARES PREDICTION OF THE MINISTRY’S RESERVATION VALUE GIVEN AN OPTIMAL UPSET OF 70 PERCENT**

Area	Reserve Value as Percent of Appraisal, Assuming Optimal 70% Upset	Bootstrap Standard Error	Bootstrap 95% Confidence Interval (Percentile Method)
1	55.8	5.0	(44.4, 66.5)
2	53.6	3.3	(47.1, 59.7)
3	55.3	1.4	(52.1, 57.9)
4	51.8	3.3	(45.0, 57.7)
6	54.7	5.3	(42.8, 65.3)

As Table 5 indicates, we estimate that an optimal upset of 70 percent implies that the Ministry’s reservation value is between 51.8 percent and 55.8 percent of the appraisal.<sup>14</sup> These estimates are less precise than the estimates presented in Table 4. The bootstrap standard errors are at least twice as large, and the bootstrap 95-percent confidence intervals in some cases exceed 20 percentage points. The lower amount of precision stems from the out-of-sample prediction of the left-hand side of equation 6. The natural variation in the shape of this expression, induced by re-sampling, is magnified by our linear regression model.<sup>15</sup> Put differently, small changes in the estimated quadratic equation are magnified when we predict far away from the observed data. Figure 6 graphically illustrates this point.

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14. We cannot estimate the reserve value for Area 10 at an upset of 70 percent because, even after extrapolation, the left-hand side of equation 6 is not increasing monotonically in  $r$ . The lowest upset for which we can estimate the reserve value is 78 percent, which yields an implicit reserve value of 56.4 percent of the appraisal.

15. For a discussion of bootstrap terminology, see Section 2 of the Appendix A.

**FIGURE 6. MAGNIFICATION OF ERROR FROM OUT-OF-SAMPLE PREDICTION OF THE RESERVATION RATE**

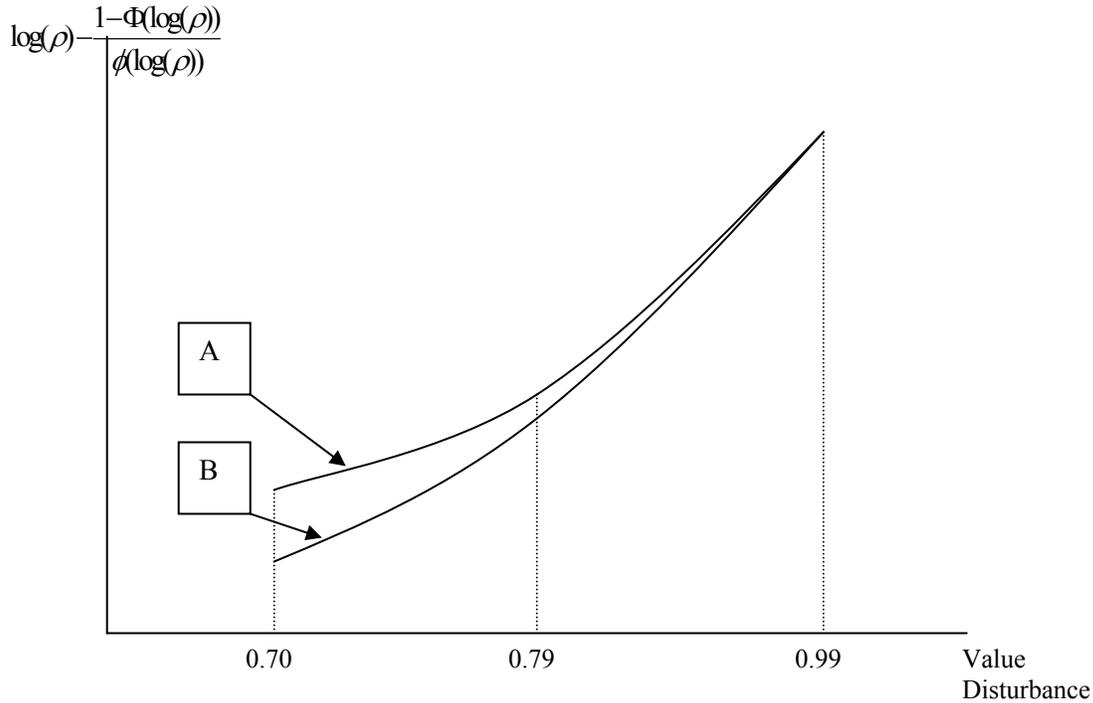


Figure 6 presents two (hypothetical) quadratic lines from bootstrap samples. Both these lines (called A and B) were estimated using data samples from pseudo value disturbances between 0.79 and 0.99. The two regression lines closely match one another for value disturbances in that range, but when those equations are used to predict reservation rates for a value disturbance of 0.70, the difference between the two regression lines increases. Thus, larger variation in the estimate of the reservation value from replication to replication occurs when those data are used to make predictions for out-of-sample reservation values. The increased variation in the estimated reservation rate will increase the standard errors of that point estimate, and therefore the confidence intervals surrounding that point estimate will rise. Thus, the level of precision of the estimated reservation rate decreases when that rate is estimated from out-of-sample predictions.

### 3.3 Potential revenue loss if the upset is set incorrectly

Using the above methodology, we can estimate the lost revenue, in percentage terms, from setting an upset of 80 percent, 85 percent, or 90 percent when the optimal upset is in fact 79 percent, given the Ministry's reservation value (opportunity cost of auctioning the timber). To perform this calculation, first assume that the optimal upset is 79 percent and consult equation 6 to find the opportunity cost,  $v_0$ , that would command such an upset rate. We then use an upset of 80 percent and the  $v_0$  implied by the optimal 79 percent rule to determine the revenue that the Ministry would experience for the average stand from equation 2. After calculating the revenue from equation 2, we can take the difference, in percentage terms, to find the rate of lost revenue from the sub-optimal upset. Below, we show the rate of revenue loss by setting an upset of between 80 and 90 percent when then optimal upset is 79 percent.

**TABLE 6. RATE OF REVENUE LOSS WHEN THE UPSET IS CHOSEN INCORRECTLY**

Area	Percent Reduction in Revenue at 80% Upset, Assuming Optimal 79% Upset	Percent Reduction in Revenue at 85% Upset, Assuming Optimal 79% Upset	Percent Reduction in Revenue at 90% Upset, Assuming Optimal 79% Upset
1	0.5	3.5	8.3
2	0.8	4.9	9.0
3	0.5	4.0	8.2
4	0.5	5.1	10.2
6	0.6	5.4	9.1
10	0.2	2.1	7.2

When the upset is off by 1 percent, the estimated revenue loss is between 0.2 percent and 0.8 percent. This revenue loss is negligible. When the upset is off by 11 percent, however, we predict revenue losses that are more substantial. The smallest revenue loss we predict is 7.2 percent, while the largest is 10.2 percent. This estimation shows the importance of understanding the opportunity cost of auctioning the timber in each district. The higher is the value that the Ministry places on that timber, the higher should be the upset price, but note that for reasons we expand on below, the timber should not be auctioned at upset prices below the Ministry's value of the timber. Finally, we cannot directly predict the revenue loss if the upset is currently too high, because we cannot directly retrieve the bid distribution below the 79 percent upset. However, using least squares to predict the revenue loss when the optimal upset is 70 percent but the upset is incorrectly set at 75 percent, we estimate that the Ministry will lose between 0 percent and 3.0 percent of revenues depending on the area in question.<sup>16</sup> If the optimal upset is 70 percent, but the upset is set at 85 percent, we estimate that revenue losses will vary between 5.4 percent and 11.1 percent.<sup>17</sup>

#### **4 THE OPPORTUNITY COST OF AUCTIONING TIMBER, AND CHANGING THE UPSET RATE**

The opportunity cost of auctioning timber is a key parameter when setting an upset rate to maximize the auction proceeds. For this reason the Ministry should obtain a reasonable estimate of its value of the timber relative to the private sector appraisal, and in all likelihood should not consider changing the upset rate until an accurate estimate of this value is determined. Once the Ministry's opportunity cost of auctioning timber is estimated, better data is collected (as we described above), and the estimation is refined, changing the upset may be in order. We discuss some examples below, where raising or lowering the upset may be appropriate.

16. This range excludes Area 10 because, as mentioned earlier, we cannot calculate the reserve value for upsets of less than 78 percent.

17. This range also excludes Area 10. At an upset of 90 percent, the marginal revenue loss in area 10 is 7.5 percent, assuming the optimal upset is 78 percent.

When considering the appropriate opportunity cost of auctioning timber, the Ministry must first realize that its opportunity cost of auctioning the timber should be less than its appraisal of that timber's value. In particular, the appraisal process is the Ministry's attempt to estimate the value of the timber stand to the private sector. However, the Ministry presumably cannot harvest the timber itself more efficiently than the private sector, so the value of the stand arises from the value of retaining the timber, either indefinitely or for future resale.

When a tract does not sell at auction, the Ministry acquires certain information. If bidders' values for a particular tract are forever constant, and if the bidders are bidding under the equilibrium described above, then the Ministry learns that the value of the tract is less than the upset it set. In other words, the expected resale value of tracts that do not sell at a given upset cannot be greater than the upset itself. Alternatively, if some components of bidder values are transient (for example, bidders' values for a particular tract depend on their current access to other, similar tracts), then a tract that fails to sell today might sell for a future price that is greater than today's upset price. Overall, the expected future resale value of a tract that fails to sell today depends on the correlation between today's values and tomorrow's values by firms. Such a correlation cannot be easily estimated from the existing data, because a large sample of tracts that failed to sell initially and were offered again later would be required. One factor that could lead to correlation in values over time (conditional on the appraised value) is that bidders may measure certain features of tracts that they consider to be important in that tract's valuation. The Ministry, however, may be unable to include all of these characteristics in its appraisal. If bidder values are correlated over time, little is gained by setting a high upset price and continually reoffering a tract in the hopes it will sell at that high price. Further, holding multiple auctions for a single tract utilizes resources from both the Ministry and the bidders that investigate the auctions. The issues associated with the dynamics of optimal upset pricing have received little attention in the economics literature to date, and a full analysis of them is beyond the scope of this paper. Such an analysis would require an analysis of the extent to which a bidder values its option of bidding on a stand in the future if that stand were to go unsold today.

Between January 1, 1999 and December 31, 2000 the Ministry auctioned 476 stands. Of those 476 stands, 51 stands or 10.7 percent received no bids, and 37 stands or 7.8 percent received only one bid. Also, 172 stands or 36.1 percent received winning bids above the Ministry's appraised value. These statistics indicate that increasing the upset rate should be done with caution, because a significant increase in the number of unsold stands could result. With fewer stands sold at auction, the pricing information from those marginal stands to the Ministry's pricing equation would diminish.

Now consider the decision to change the upset rate. The preceding analysis suggests a procedure for optimally setting the upset that can be implemented by the BC government once better data is available. The first step is to estimate the optimal upset and its associated confidence interval. The government can begin by calculating  $v_0$  relative to its appraisal of the timber's worth on a district-by-district, or area-by-area basis. It can then employ the estimation procedure used in this paper to find the optimal upset equation and use the estimated reservation value,  $v_0$ , to find the optimal upset via this equation. Finally, the government can use the bootstrap to find a 95 percent confidence interval for this estimate.

The second step is to determine whether the prevailing upset is sufficiently close to the optimal upset in each area. The prevailing upset might not be considered optimal if (1) it does not lie within the 95 percent confidence interval of the estimated optimal upset in numerous areas and (2) it significantly reduces revenue relative to the optimal upset in those areas. If both of these conditions are met, the Ministry should consider changing the upset rate. If only condition (1) is met, then the economic benefits of changing the upset may be relatively small, and the government should seriously consider whether the benefits of adjustment outweigh the costs.

Also, we must consider that a single upset rate is set throughout the province. Thus, we must understand the economic significance that a provincial upset rate may have on individual districts or areas. Particular attention should be paid to the effects of adjusting the upset on total revenues. For example, setting the upset optimally for a group of districts representing 20 percent of revenues could increase revenues 10 percent in these districts while reducing revenues by 5 percent in all other districts. This particular change in upset policy, while optimal for some districts, actually reduces total revenues.

## 5 CONCLUSION

The upset price plays a critical role in the BC timber auctions. It serves to enhance revenues in situations where competition is weak, to limit the incentive for collusive bidding, and to provide information to bidders. Our analysis of the data suggests that an upset of about 70 percent (a rollback of 30 percent) maximizes auction revenues when the Ministry values timber at approximately 52 percent to 56 percent of its appraised value. In arriving at this number, the analysis makes the following assumptions:

- The bidders do not collude.
- The bidders' values are drawn independently from the same probability distribution in a given area.
- The bidders bid to maximize their expected profits from the auction.
- The number of potential bidders does not depend on the upset.

These assumptions are standard in the literature and are a good first approximation. Still, given the importance of setting an appropriate upset, we urge the BC government to refine its estimate of the upset as better data becomes available. An improved dataset would include all the observed bids, not just the winning bid, and a better understanding of the Ministry's opportunity cost of auctioning the timber.

We have assumed that the BC government's objective is to maximize auction revenues. If instead, the goal is efficiency (maximization of the gains from trade) and collusion is not a problem, then the upset should be set at the government's reservation value for the timber. However, in situations where the reservation value is low, then setting the upset at the reservation value may create too strong an incentive for the bidders to collude, in which case raising the upset above the reservation value is justified on both revenue and efficiency grounds. On the other hand, setting a lower upset may increase bidder participation in which case setting a lower upset may increase both revenues and efficiency (see Bulow and Klemperer 1996).

## APPENDIX A. AN EXAMPLE OF THE DETERMINATION OF THE OPTIMAL UPSET

Suppose that values,  $v_i$ , are a random sample drawn from the density function  $f(v) = 1$ , on the unit interval, with corresponding distribution function  $F(v) = v$ . We conduct 200 sealed-bid auctions *without an upset price*, and four bidders submit bids in every auction (in each auction, each bidder receives a new, independent draw from the distribution). Also, assume that we only record the winning bid in each auction. Thus, our dataset includes 200 winning bids for first-price sealed bid auctions where four bidders competed. Below, we go through a nonparametric analysis of optimal upset estimation for the full sample, and then use “bootstrapping” on 500 samples of 200 observations (drawn from the original sample, with replacement) to estimate the standard error of the optimal upset.

### 1 THE NONPARAMETRIC ESTIMATION

From the statistics literature, the distribution of the highest (i.e. winning) value,  $v^{(1)}$ , has the distribution function  $F^{(1)}(v) = F(v)^N$ , where  $N$  is the number of bidders. Thus,  $F^{(1)}(v) = v^4$  in this example. We therefore sample 100 *winning* values from this distribution function, and calculate the corresponding winning bids according to the optimal bidding rule:

$b_i = s(v_i, N) = v_i - \frac{1}{F^{N-1}(v_i)} \int_{\underline{v}}^{\bar{v}} F^{N-1}(x) dx$ . Substituting  $F(v) = v$ ,  $N = 4$ , and  $\underline{v} = 0$  into the optimal

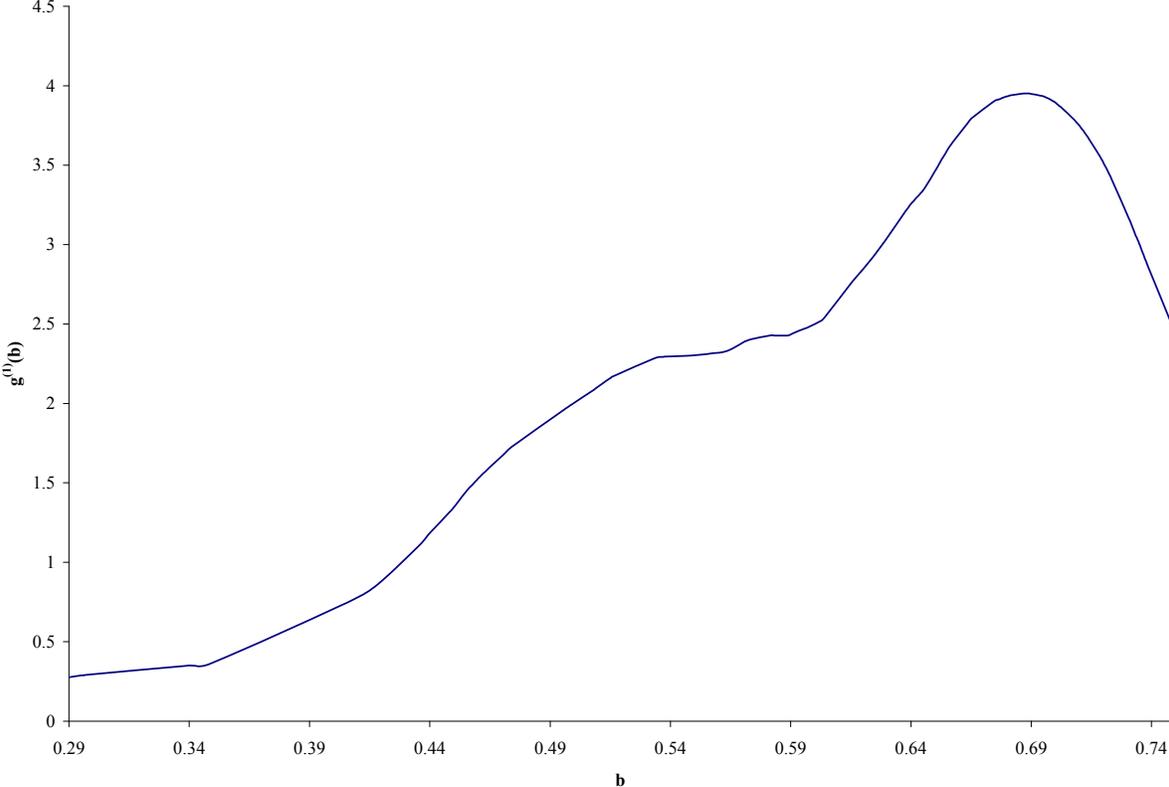
bid equation, we can simplify the equilibrium bid strategy to:

$$(A1) \quad b_i = s(v_i, 4) = \frac{3v_i}{4}$$

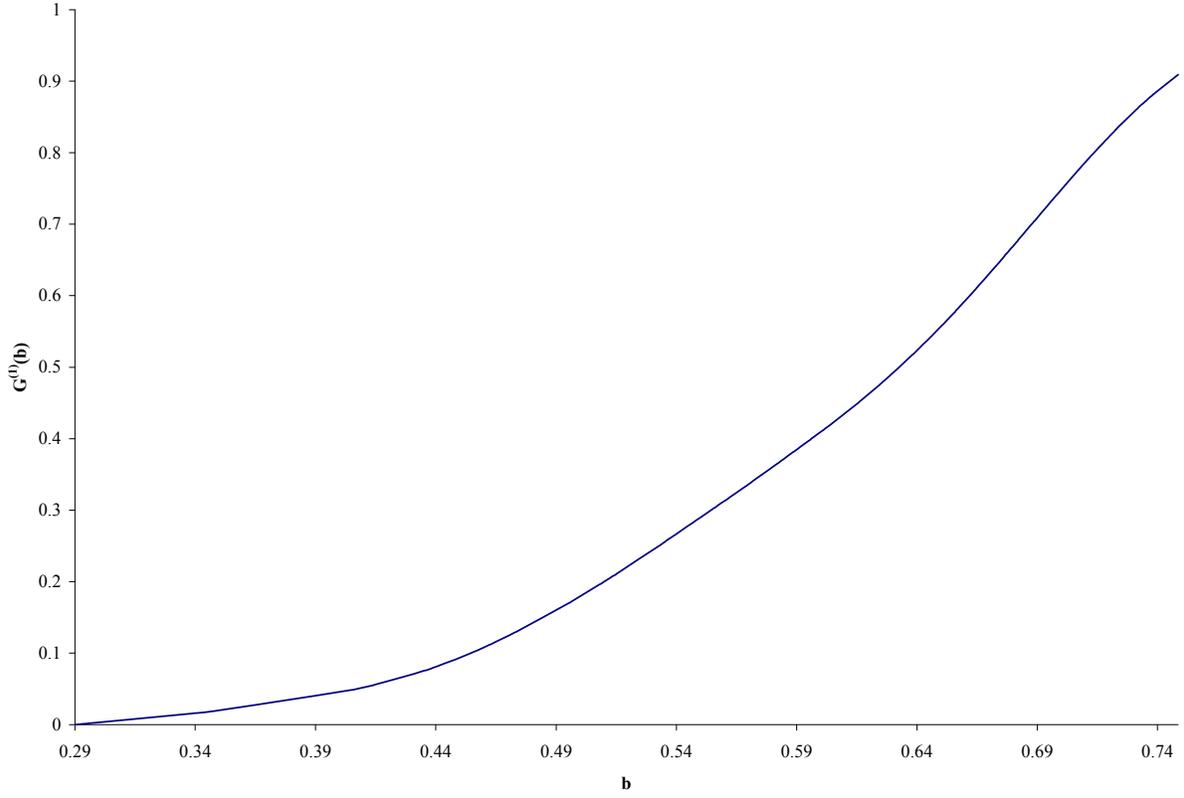
Thus, we transform our values into bids according to equation A1. As researchers, we pretend that the only observables we have are the winning bids that we generated above, and the number of bidders in each auction, which is four.

We are now ready to perform the steps of the nonparametric estimation to determine the optimal upset. The first step of this technique is to run a kernel density estimate on the observed bids to obtain an estimate of  $g^{(1)}(b)$ , the density of the winning bids. This estimation yields the density function depicted in Figure A1, and numerical integration of the estimated density yields the winning bid distribution shown in Figure A2.

**FIGURE A1. KERNEL DENSITY ESTIMATES OF THE WINNING BID DENSITY**



**FIGURE A2. ESTIMATE OF THE WINNING BID DISTRIBUTION**



The kernel density estimate of  $g^{(1)}(b)$  seems reasonable, as most of its mass occurs for higher bids, as we would expect. The estimated winning bid distribution also appears realistic. It has the proper shape—that is, it entirely lies below the 45-degree line.

After obtaining estimates of the density and distribution of the winning bids, we now apply the inverse bid function to obtain estimates of the corresponding values—called pseudo values in the literature. This step is done by applying equation (A2)<sup>18</sup> to the observed bids, densities, and distributions.

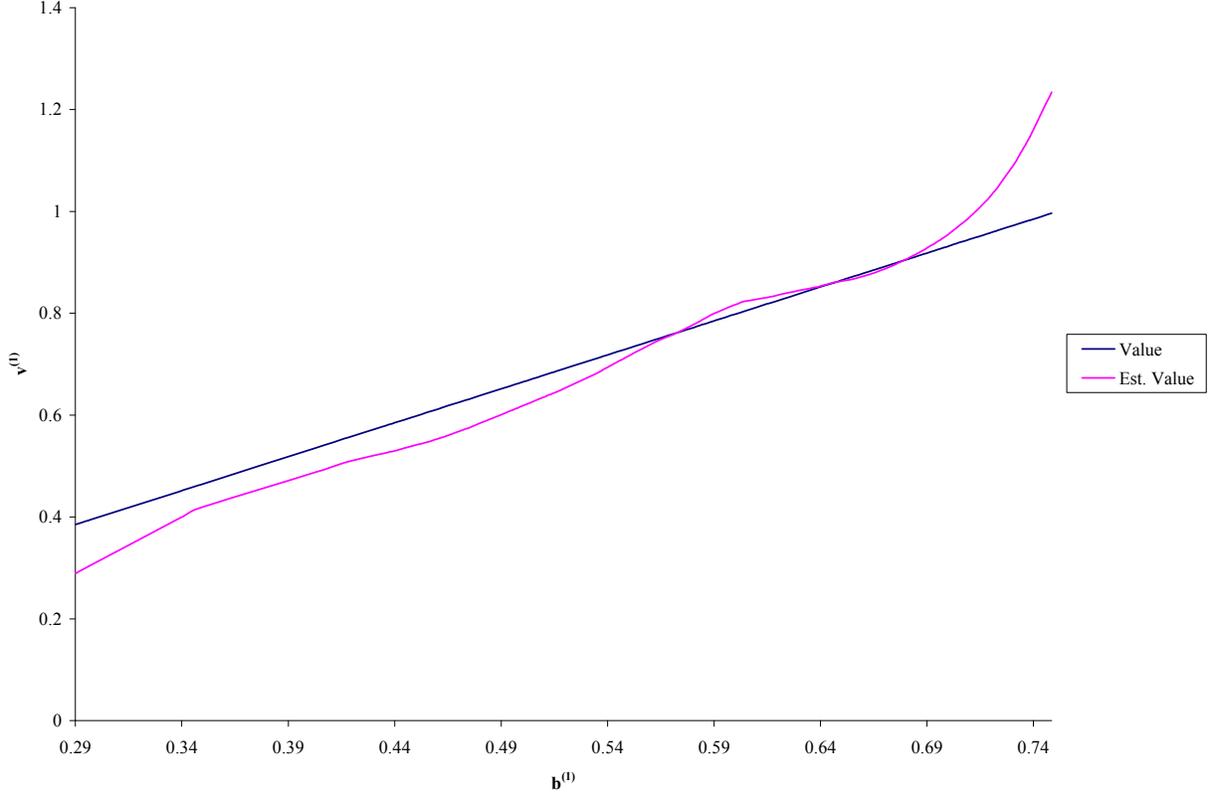
$$(A2) \quad \hat{v}_i^{(1)} = b_i^{(1)} + \frac{N}{N-1} \frac{\hat{G}^{(1)}(b_i^{(1)})}{\hat{g}^{(1)}(b_i^{(1)})}$$

In equation A2,  $\hat{G}(\cdot)$  and  $\hat{g}(\cdot)$  are the distribution and density estimates, respectively, from the kernel procedure. We now have estimated our bid function, because we have both actual bids and estimates of the values that generated those bids. Figure A3 shows the estimated bid function, and how it compares to the true bid function.

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18. This equation is given on page 27 of ELLV (1995). It can be derived directly from the bidder's maximization problem, as described in the text.

**FIGURE A3. ESTIMATED AND ACTUAL INVERSE BID FUNCTIONS**



In Figure A3, we note that our estimation of the true bid function is precise, with the exception of the upper tail. That we have difficulty estimating the tail is little surprise given the discussion above. Nevertheless, the kernel procedure performs well in this example, as we would hope.

After we have calculated a pseudo-value for each bid in the dataset, we then use kernel estimation once again to obtain an estimate of the density of those pseudo-values. The distribution can be constructed using numerical integration. As a final step before determining the optimal upset, we must transfer our estimates of the distribution and density of the highest value,  $\hat{F}^{(1)}(\cdot)$  and  $\hat{f}^{(1)}(\cdot)$  respectively, into estimates of the population distribution and density—namely,  $\hat{F}(\cdot)$  and  $\hat{f}(\cdot)$ . This transformation is performed according to equations A3 and A4, below. Equation A3 is simply the inverse of the distribution of the highest ordered value in auctions of five bidders:  $F^{(1)}(\cdot) = F(\cdot)^5$ , and equation A4 is calculated by differentiating equation A3 with respect to  $v$ .

$$(A3) \quad \hat{F}(\cdot) = \hat{F}^{(1)}(\cdot)^{1/5}$$

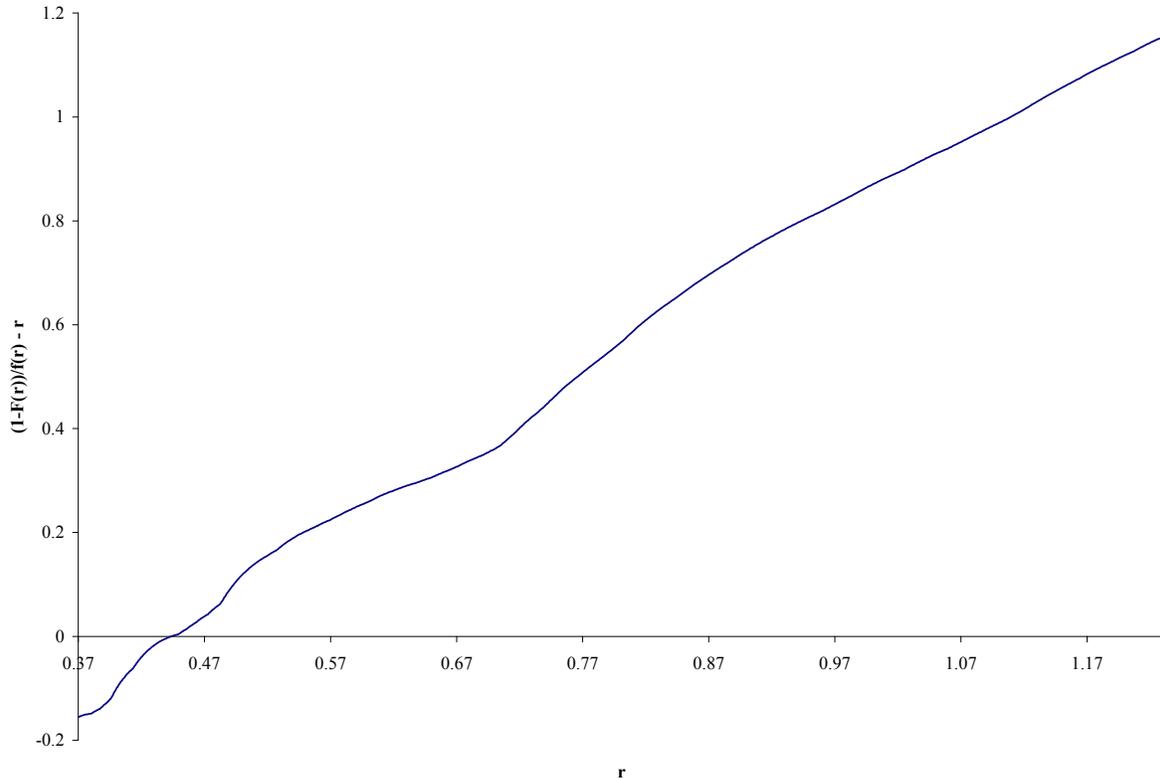
$$(A4) \quad \hat{f}(\cdot) = \frac{1}{5} \hat{f}^{(1)}(\cdot) * \hat{F}^{(1)}(\cdot)^{-4/5}$$

Finally, we are ready to estimate the optimal upset. Recall from equation 3, that with  $v_0 = 0$ , the optimal upset solves the following equation:

$$(A5) \quad \frac{1 - \hat{F}(r)}{\hat{f}(r)} - r = 0$$

In our dataset, we generate a function identical to that in equation (A5) using the estimates of the density and distribution functions,  $\hat{f}(r)$  and  $\hat{F}(r)$  respectively. Figure A4 graphs this function on the domain where it is monotone increasing.

**FIGURE A4. OPTIMAL UPSET EQUATION**



The upset price at which the optimal upset equation holds—that is, where the line intersects the horizontal axis—is the optimal upset price. In this example, the optimal upset price is approximately 0.443, which is close to the true optimal upset price of 0.500. To better determine the precision of our estimated optimal upset price is to the true optimal upset price, we need to calculate the standard error of the estimate.

## 2 ESTIMATING THE STANDARD ERROR OF THE OPTIMAL UPSET THROUGH BOOTSTRAPPING

Bootstrapping is a technique that, roughly speaking, samples many times from the original sample to determine the variance of a consistent estimator.<sup>19</sup> Intuitively, it works by calculating how the estimates change when the sample changes. Bootstrapping is likely the simplest method for calculating the standard error of the estimated optimal upset price in this application.

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19. For a more thorough explanation of bootstrapping, see Efron and Tibshirani (1993) or Greene (1997).

We implement the bootstrap as follows. First, define our estimator of the optimal upset price as  $\hat{r}_o$ . That is, our estimator,  $\hat{r}$ , was run on a sample of 200 observations. We then randomly select a new sample of size,  $m$ , from the 200 observations, and calculate  $\hat{r}_m$ . The trick to the sampling methodology is that it is performed *with replacement*. To give an example, suppose we choose  $m$  equal to 50. We construct the sub-sample by choosing a single observation from the 200 observation sample, recording that observation, replacing it, and then sampling again 49 more times. Thus, our 50 observations will contain duplicates of some of the original observations, and will omit others. Then, after generating each sample, we calculate  $\hat{r}_m$  for that sample.

The choice of  $m$  depends on many factors. One common choice is to set  $m$  equal to the original sample size—in this case, 200. This choice is sensible because the estimates obtained from the bootstrap sample will have similar properties to estimates obtained from the original sample.

Now, let  $I$  denote the total number of bootstrap replications—that is, the number of times we draw a new bootstrap sample. Because our estimator does not necessarily have a normal, or even a symmetric, distribution, the most straightforward way to calculate a 95 percent confidence interval is to calculate a range of values that contains 95 percent of the bootstrap estimators  $\hat{r}_m$ . That is, we calculate the .025 and .975 quantiles from the empirical distribution of  $\hat{r}_m$ , that is, the value of the estimate  $\hat{r}_m$  with rank .025  $I$  and the value of the estimate with rank .975  $I$ .

An alternative method, analogous to standard variance calculations, is to provide a direct estimate of the variance according to equation (A6) (see, e.g., Efron (1979)). When  $m$  is equal to the original sample size, we have:

$$(A6) \quad \text{AsymVar}[\hat{r}] \approx \frac{1}{I} \sum_{i=1}^I [\hat{r}(i)_m - \hat{r}_o]^2$$

Using (A6), we can then calculate a 95 percent confidence interval for the upset price estimator.

To estimate the 95-percent confidence interval accurately using the percentile method, one thousand bootstrap replications is sufficient.<sup>20</sup> However, the optimal upset equation does not yield an optimal upset in every replication. For certain samples, the left-hand side of the equation never equals zero. In our bootstrap there is only a single replication that did not yield an optimal upset. We assign a value of zero to the observation for that replication, as would be done by the seller. We then calculate a mean, standard error, and confidence intervals for the optimal upset estimator, as shown in Table A1.

**TABLE A1. SUMMARY STATISTICS FOR THE OPTIMAL UPSET ESTIMATOR**

<b>Point Estimate</b>	<b>Standard Error</b>	<b>95% Confidence Interval: Percentile Method</b>	<b>95% Confidence Interval: Variance Method</b>
0.473	0.035	(0.421, 0.546)	(0.405, 0.541)

The estimated optimal upset is within two standard deviations of the true optimal upset, which is 0.500. The true optimal upset price is also contained by both 95 percent confidence intervals. Thus, the

20. STATA CORP, STATA REFERENCE MANUAL A-G: RELEASE 7 165 (2001).

estimated optimal upset is not statistically different from the true optimal upset price at the 5-percent level of significance, meaning that the nonparametric estimation technique performs reasonably well when it is applied to data generated from the benchmark auction case.

It is important to note that smaller sample sizes lead to less accurate bootstrap estimates of the parameter of interest. In other words, using a sample containing fewer than 200 observations would yield a point estimate of the optimal upset farther from its true value, larger standard errors, and wider confidence intervals. Thus, one must be cautious when interpreting parameter estimates from a bootstrap on smaller samples.

We now consider any bias in our estimator. To evaluate the bias, we use Monte Carlo simulations. In particular, we perform the following procedure 100 times: we generate a new dataset with 200 independent observations. Each new dataset is independent of the others. We find 99 datasets from which we are able to calculate a nonzero optimal upset. Thus, we use the 100 estimated optimal upset prices to compute the mean estimate, the standard error of the mean, and the bias. Our results are presented in Table A2.

**TABLE A2. MONTE CARLO ANALYSIS OF THE BIAS OF THE OPTIMAL UPSET ESTIMATOR**

<b>Mean Estimate</b>	<b>Standard Error of Estimate</b>	<b>Bias</b>
0.496	0.069	-0.004

Our results indicate that, on average, the estimation performs well for sample sizes of 200. The bias from the optimal upset calculation is very small, and the standard error of the upset calculation is reasonable. We find that a 95 percent confidence interval for the optimal upset calculation is (0.361, 0.631), which includes the true upset of 0.5.

## APPENDIX B. METHODOLOGY AND RESULTS OF THE KERNEL ESTIMATION PROCEDURE

We begin by maintaining the assumptions and notation discussed above. In particular, assume that values are distributed identically, and are independent draws. Therefore, values are a random sample. Let  $v_i$  denote the observed value of bidder  $i$ , consistent with notation above. Also,  $f(v)$  and  $F(v)$  represent the density and distribution functions, which are defined on support  $[\underline{v}, \bar{v}]$ , for the underlying sample of values. Now define  $l = \{1, 2, \dots, L\}$  to be an index of the auction lot, and assume that  $f(\cdot)$  and  $F(\cdot)$  apply to all lots. Thus,  $N_l$  denotes the number of bidders willing to bid in lot  $l$ , if there was not a upset price. Letting  $r_l$  denote the upset price in lot  $l$ , the optimal bid strategy of bidder  $i$ , for bids greater than the upset, is

$$(B1) \quad b_{li} = s(v_{li}, N_l, r_l) = v_{li} - \frac{1}{F^{N_l-1}(v_{li})} \int_{r_l}^{\bar{v}} F^{N_l-1}(x) dx$$

$$b_{li} = 0, \text{ if } b_{li} < r_l$$

Because the right-hand side of equation B1 is increasing, the density and distribution functions of the bids,  $g(b)$  and  $G(b)$ , respectively, are increasing as functions of  $f(\cdot)$  and  $F(\cdot)$ . In particular,  $F(v) = G(s(v))$ , and differentiating yields the result  $f(v) = g(b)s'(v)$ . GPV (1995) show that these transformations can be substituted into the first-order condition for bidder  $i$ , in an auction with upset price of  $r$ , yielding the inverse bid function

$$(B2) \quad v_i = b_i + \frac{1}{N-1} \frac{\bar{G}(b_i)}{\bar{g}(b_i)} + \frac{1}{N-1} \frac{F(r)}{1-F(r)} \frac{1}{\bar{g}(b_i)}.$$

In equation (B2), we use  $\bar{G}(b_i)$  and  $\bar{g}(b_i)$  to represent the truncations of  $G(\cdot)$  and  $g(\cdot)$  at the upset price. This is a necessary adjustment, because given a upset price of  $r$ , only bidders with values at least as big as  $r$  will submit bids at auction. But equation B2 presents us with two complications. First, we do not observe  $N$ , the actual number of bidders. Rather, we observe  $\bar{N}$ , which denotes the number of bidders who placed bids above the upset. But GPV (1995) suggest an estimator for the actual number of bidders as the maximum number of observed bidders within a reasonable group of lots. In the Interior, an obvious extension would be to calculate, by district, the maximum number of observed bids, and use this as an estimate for the actual numbers of bidders, for any given lot, in that district. Formally, let  $K$  represent the set of auction lots that occurred within ‘‘district  $k$ .’’ Then, our estimate of the number of bidders in district  $k$ ,  $\hat{N}_k$ , is

$$(B3) \quad \hat{N}_k = \max_{l \in \{K\}} \bar{N}_l.$$

Next, we must estimate  $F(r)$ , which we cannot directly estimate from the bid data. To estimate  $F(r)$ , GPV (1995) suggest the use of a nonparametric regression to estimate the expectation of the observed number of bidders. In particular, the expected number of observed bidders will equal that number of potential bidders multiplied by the probability that any bidder has value above the upset price. Formally, this is written as  $E[\bar{N} | r] = N[1 - F(r)]$ . This equation can be rewritten to yield

$$(B4) \quad F(r) = 1 - \frac{E[\bar{N} | r]}{N}.$$

We can substitute our estimate of  $N$  for the  $k^{\text{th}}$  district,  $\hat{N}_k$ , into equation (B4), and then use a nonparametric regression of  $r$  on  $\bar{N}$  to complete the estimation of equation (B4). Substituting this information into equation (B2) then allows us to obtain estimates for the values,  $\hat{v}$ , which we can use to estimate the underlying distribution of values.<sup>21</sup>

A final complication to the substitution of the estimates into equation (B2) is that we only observe the winning bids. Thus, what we are really estimating in equation (B2) is

$$(B5) \quad v_i^{(1)} = b_i^{(1)} + \frac{N}{N-1} \frac{\bar{G}^{(1)}(b_i)}{\bar{g}^{(1)}(b_i)} + \frac{N}{N-1} \frac{F(r)}{1-F(r)} \frac{1}{\bar{g}^{(1)}(b_i)},$$

where  $\bar{g}^{(1)}(\cdot)$  and  $\bar{G}^{(1)}(\cdot)$  are the density and distribution functions of the highest ordered bid, given that bid is above the upset price. Also,  $v_i^{(1)}$  represents the highest ordered value, with corresponding bid  $b_i^{(1)}$ . Thus, our value estimates reflect the highest ordered values in the distribution, given that those values were above the upset price. Formally, we write this distribution as

$$(B6) \quad \bar{F}^{(1)}(x|r) = \Pr(\text{all } v_i < x | \bar{N} \text{ of the } v_i > r).$$

To complete the estimation, we must express equation (B6) in terms of the bid distribution  $F(\cdot)$ , so that it may be retrieved from our estimates. To accomplish this goal, first consider the distribution of the highest order statistic from  $N$  random draws from the distribution  $F(\cdot)$ . This distribution is  $F^{(1)}(v) = F(v)^N$ . Notice that the order statistic from equation (B6) can be thought of as the highest ordered bid in a random sample of  $\bar{N}$  bids, all drawn from the distribution  $F(\cdot)$ , truncated at the upset. Specifically, the  $\bar{N}$  draws were a random sample from the distribution  $F(v|v > r) = \frac{F(v) - F(r)}{1 - F(r)}$ . Combining this result with the definition of an order statistic yields

$$(B7) \quad \bar{F}^{(1)}(v) = \left( \frac{F(v) - F(r)}{1 - F(r)} \right)^{\bar{N}}.$$

Given that  $\bar{N}$  is observed in the SBFEP dataset, and  $1 - F(r)$  was estimated via the nonparametric regression described above, we can solve for  $F(\cdot)$ .

## 1 An extension using bid disturbances

Given the discussion above, consider now an extension where we believe that each auction lot has a common value component that all bidders realize. For the B.C. timber auctions, this common component could be the Ministry's appraised value. In addition to the common component, there is a stochastic component of bidder valuations that drives the randomness that we observe in the underlying values and therefore the bids. Formally, suppose that values are generated according to equation B8.

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21. An alternative, and simpler approach, would be to realize that the upset is really constant at 70 percent throughout any given district, and therefore,  $F(r)$  should be constant within each district as well. To estimate a constant  $F(r)$ , we could take the total number of observed bids in all auctions within a district, and divide by the total number of potential bids. This calculation would yield  $1 - F(r)$ , and we could then solve for  $F(r)$ .

$$(B8) \quad v_{il} = v_l + \varepsilon_i$$

The index  $l$ , which varies from 1 to  $L$ , represents indicates the appropriate auction lot. The index  $i$  indicates the bidder as before. The variable  $v_l$  is therefore a common value component imbedded in each lot  $l$  bidder's private value. The error  $\varepsilon_i$  is drawn from a random sample, and the underlying density is the same for all  $i$  and  $l$ . Thus, any stochastic behavior in the bids is attributed entirely to  $\varepsilon_i$ , and that stochastic behavior is consistent across bidders and auctions. Crucially, we assume that the value  $v_l$  is observed by all bidders, while  $\varepsilon_i$  is private information for bidder  $i$ .

Moving forward, define the density and distribution functions of  $\varepsilon_i$  as  $\varphi(\cdot)$  and  $\Phi(\cdot)$  respectively, and recall that the density and distribution functions of the values were  $f(\cdot)$ , and  $F(\cdot)$ . From equation B8 above we can write:

$$(B9) \quad F(v) = \Pr\{v_{il} < v\} = \Pr\{v_l + \varepsilon_{il} < v\} = \Pr\{\varepsilon_{il} < v - v_l\} = \Phi(v - v_l)$$

Equation (B9) shows that  $\Phi(\cdot)$  is obtained as an additive shift (of magnitude  $v_l$ ) of  $F$ . This implies that we can use  $\Phi(v - v_l) = \Phi(\varepsilon)$  interchangeably with  $F(v)$  in our estimation, and obtain identical results. Thus, we can write our bid equation as follows:

$$(B10) \quad b_{li} = s(v_{li}, N_l, r_l) = v_l + \varepsilon_i - \frac{1}{\Phi^{N_l-1}(\varepsilon_i)} \int_{r_l}^{\bar{\varepsilon}} \Phi^{N_l-1}(x) dx$$

$$b_{li} = 0, \text{ if } b_{li} < r_l$$

Examination of (B10) yields the attractive result that  $v_l$  is the non-stochastic component of the bids as well as the values. For simplicity, we rewrite the bid function as  $b_{li} = s(v_{li}, N_l, r_l) = v_l + u_i$ , where  $u_i$  is the stochastic component of the bids. Because the bid function is monotonic, we still have the transformation between  $G(\cdot)$  and  $F(\cdot)$ , and therefore  $G(\cdot)$  and  $F(\cdot)$  and  $\Phi(\cdot)$ . Thus, we can tailor our estimation to this special case, by first estimating the non-stochastic effect, and applying the nonparametric estimation to the residual. We let  $\bar{\Theta}(\cdot)$  and  $\omega(\cdot)$  denote the distribution and density functions the random variables  $u_i$ , and note that  $\bar{\Theta}(\cdot)$  will transform directly into  $G(\cdot)$  and  $F(\cdot)$  by subtracting the non-stochastic term  $v_l$  from the bids.

We can first obtain the residuals.<sup>22</sup> The difference between the residuals and the observed bids (i.e. the regression prediction) is the non-stochastic component of the values. We can then use the kernel density procedure described above on the residuals to predict density and distribution functions of the bids. We then back out the pseudo values according to equation B11 below:

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22. In our analysis above, we used the appraisal as the common component of the value, and the deviation from that appraisal as the disturbance. Another methodology would be to run a least squares regression of the observed bids on certain stand characteristics, and obtain the regression residuals as the bid disturbances.

$$(B11) \quad v_i^{(1)} = b_i^{(1)} + \frac{N}{N-1} \frac{\bar{\Theta}^{(1)}(u_i)}{\bar{\omega}^{(1)}(u_i)} + \frac{N}{N-1} \frac{F(r)}{1-F(r)} \frac{1}{\bar{\omega}^{(1)}(u_i)}$$

A transformation similar to equation B7 above then completes the procedure.

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