The Optimality of Being Efficient

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Common Reaction

Why worry about ***efficiency***,
when there is resale?

Our Conclusion

Why worry about
revenue maximization,
when there is resale?

Standard auction literature

n bidders; one or more objects;
no resale.

This paper

n bidders; one or more objects;
perfect resale.

Outline

- Examples
- Incentive to misassign the good
  - Identical objects model
  - Optimal auction
- Optimal auctions recognizing resale
- An efficient auction
  - Is optimal with perfect resale
  - Can be implemented with a Vickrey auction with reserve pricing
- Seller does strictly worse by misassigning goods
- Applications
  - Treasury auctions
  - IPOs

Examples

Example 1
- One object
- Two bidders w/ private values
- Strong's value is uniform between 0 and 10
- Weak's value is commonly know to be 2

Figure 1. Alternative assignment rules (Weak's value = 2)

<table>
<tr><td>Efficient auction</td><td>Weak</td><td colspan="3">Strong</td></tr>
<tr><td>Optimal auction</td><td colspan="3">Weak</td><td>Strong</td></tr>
<tr><td>Resale constrained</td><td colspan="2">None</td><td colspan="2">Strong</td></tr>
<tr><td>Strong's value</td><td>0</td><td>2</td><td>5</td><td>6 … 10</td></tr>
</table>

Examples

Example 2
- One object
- Two bidders w/ independent private values
- Strong has value v_H or v_M
- Weak has value v_H or v_L
- $v_L < v_M < v_H$

Example

Optimal auction, for some parameter values, takes the form:

		Weak	
		v_H	v_L
Strong	v_H	Either Bidder	Strong
	v_M	Weak	Weak

Example

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		Weak	
		v_H	v_L
Strong	v_H	Either Bidder	Strong
	v_M	Weak	~~Weak~~ Strong

Now introduce sequential bargaining as a resale mechanism, following the auction. Whenever Weak suboptimally wins the good, his value is v_L , and Strong's value is v_M. Can trade at (v_L+v_M) / 2. Inefficient allocation is undone!

Seller's revenues are strictly suboptimal (Theorem 5)

Identical Objects Model

- Seller has quantity 1 of divisible good (value = 0)
- n bidders; i can consume $q_i \in [0,\lambda_i]$
  $q = (q_1,\dots,q_n) \in Q = \{q \mid q_i \in [0,\lambda_i] \ \& \ \Sigma_i q_i \leq 1\}$
- t_i is i's type; $t = (t_1,\dots,t_n)$; $t_i \sim F_i$ w/ pos. density f_i
- Types are independent
- Marginal value $v_i(t,q_i)$
- i's payoff if gets q_i and pays x_i:

$$\int_0^{q_i} v_i(t,y)\,dy - x_i$$

Identical Objects Model (cont.)

Marginal value $v_i(t,q_i)$ satisfies:

- *Value monotonicity*
  - non-negative
  - increasing in t_i
  - weakly increasing in t_j
  - weakly decreasing in q_i
- *Value regularity*: for all $i, j, q_i, q_j, t_{-i}, t_i' > t_i$,
  $v_i(t_i,t_{-i},q_i) > v_j(t_i,t_{-i},q_j) \Rightarrow v_i(t_i',t_{-i},q_i) > v_j(t_i',t_{-i},q_j)$

Identical Objects Model (cont.)

- Bidder i's marginal revenue:
  marginal revenue seller gets from awarding additional quantity to bidder i

$$MR_i(t,q_i) = v_i(t,q_i) - \frac{1-F_i(t_i)}{f_i(t_i)}\frac{\partial v_i(t,q_i)}{\partial t_i}$$

Revenue Equivalence

Theorem 1. *In any equilibrium of any auction game in which the lowest-type bidders receive an expected payoff of zero, the seller's expected revenue equals*

$$E_t\left[\sum_{i=1}^{n}\int_0^{q_i(t)} MR_i(t,y)\,dy\right]$$

Optimal Auction

- *MR monotonicity*
  - increasing in t_i
  - weakly increasing in t_j
  - weakly decreasing in q_i
- *MR regularity*: for all $i, j, q_i, q_j, t_{-i}, t_i' > t_i$,

$MR_i(t,q_i) > MR_j(t,q_j) \Rightarrow MR_i(t_i',t_{-i},q_i) > MR_j(t_i',t_{-i},q_j)$

Theorem 2. *Suppose MR is monotone and regular. Seller's revenue is maximized by awarding the good to those with the highest marginal revenues, until the good is exhausted or marginal revenue becomes negative.*

Optimal Auction is Inefficient

- Assign goods to wrong parties
  - High MR does not mean high value
- Assign too little of the good
  - MR turns negative before values do

Three Seller Programs

1. Unconstrained optimal auction

(standard auction literature)
Select assignment rule and pricing rule to

max E[Seller Revenue]
s.t. Incentive Compatibility
Individual Rationality

Three Seller Programs

2. Resale-constrained optimal auction

(Coase Theorem critique)
Select assignment rule and pricing rule to

max E[Seller Revenue]
s.t. Incentive Compatibility
Individual Rationality
Efficient resale among bidders

Three Seller Programs

3. Efficiency-constrained optimal auction

(Coase Conjecture critique)
Select assignment rule and pricing rule to

max E[Seller Revenue]
s.t. Incentive Compatibility
Individual Rationality
Efficient resale among bidders *and seller*

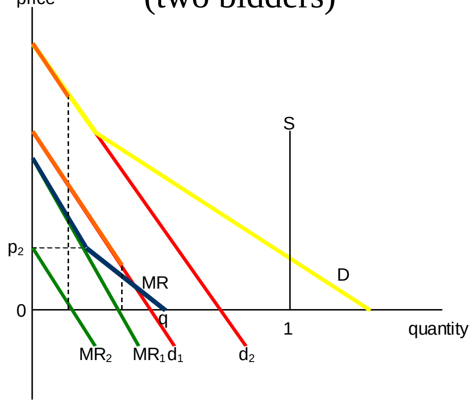
1. Unconstrained optimal auction

Select assignment rule $q(t)$ to

$$\max_{q(t)\in Q} E_t\left[\sum_{i=1}^{n}\int_0^{q_i(t)} MR_i(t,y)\,dy\right]$$

Q = {All feasible assignment rules.}

1. Unconstrained optimal auction (two bidders)



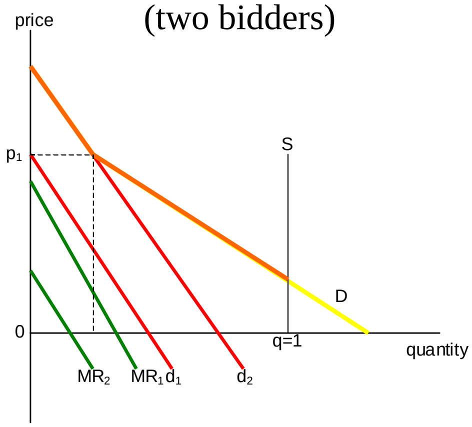
3. Efficiency-constrained optimal auction

Select assignment rule $\bar{q}^R(t)$ to

$$\max_{q(t)\in\bar{Q}^R} E_t\left[\sum_{i=1}^{n}\int_0^{q_i(t)} MR_i(t,y)\,dy\right]$$

$\bar{Q}^R$ = {Ex post efficient assignment rules.}

3. Efficiency-constrained optimal auction (two bidders)



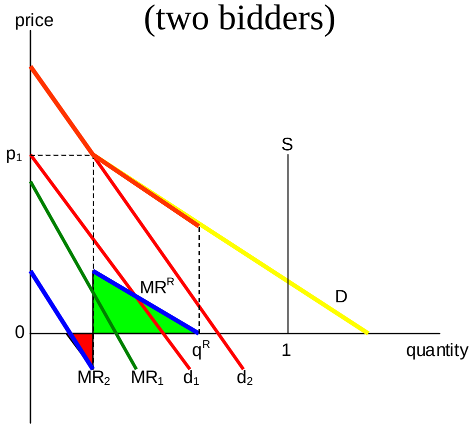
2. Resale-constrained optimal auction

Select assignment rule $q^R(t)$ to

$$\max_{q(t)\in Q^R} E_t\left[\sum_{i=1}^{n}\int_0^{q_i(t)} MR_i(t,y)\,dy\right]$$

Q^R = {Resale - efficient assignment rules.}

2. Resale-constrained optimal auction (two bidders)



Theorem 4. *In the two-stage game (auction followed by* ***perfect*** *resale), the seller can do no better than the resale-constrained optimal auction.*

Proof. Let $a(t)$ denote the probability measure on allocations ***at end of resale round***, given reports t. Observe that, viewed as a static mechanism, $a(t)$ must satisfy IC and IR. In addition, $a(t)$ must be resale-efficient. ■

Can we obtain the upper bound on revenue?

resale process is *coalitionally-rational against individual bidders* if bidder i obtains no more surplus s_i than i brings to the table: $s_i \le v(N \mid q,t) - v(N \sim i \mid q,t)$.

That is, each bidder receives no more than 100% of the gains from trade it brings to the table.

Vickrey auction with reserve pricing

Seller sets monotonic aggregate quantity that will be assigned to the bidders, an efficient assignment $q^*(t)$ of this aggregate quantity, and the payments $x^*(t)$ to be made to the seller as a function of the reports t where

$$x_i^*(t) = \int_0^{q_i^*(t)} v_i(\hat{t}_i(t_{-i},y),t_{-i},y)\,dy, \text{ where}$$

$$\hat{t}_i(t_{-i},y) = \inf_{t_i}\left\{t_i \mid q_i^*(t_i,t_{-i}) \ge y\right\}.$$

Bidders simultaneously and independently report their types t to the seller.

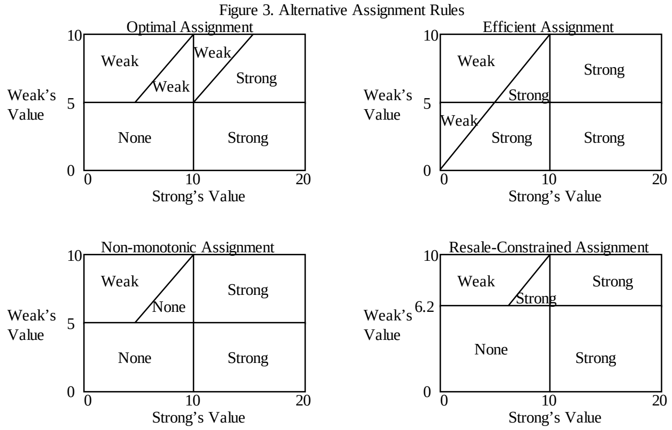
Can we attain the upper bound on revenue?

Theorem 5 (Ausubel and Cramton 1999). *Consider the two-stage game consisting of the Vickrey auction with reserve pricing followed by a resale process that is coalitionally-rational against individual bidders. Given any monotonic aggregate assignment rule , sincere bidding followed by no resale is an ex post equilibrium of the two-stage game.*

Examples

- Example 3
- Strong's value is uniform between 0 and 20
- Weak's value is uniform between 0 and 10
- $MR_s(s) = 2s - 20$
- $MR_w(w) = 2w - 10$
- Assign to Strong if $s > w + 5$ and $s > 10$
- Assign to Weak if $s < w + 5$ and $w > 5$
- Keep the good if $s < 10$ and $w < 5$

Figure 3. Alternative Assignment Rules



Can the seller do equally well by misassigning the goods?

Can the seller do equally well by misassigning the goods?

NO!

The seller's payoff from using an inefficient auction format is ***strictly less*** than from using the efficient auction.

Setup for "strictly less" theorem:

- *Multiple identical objects*
- *Discrete types*

Setup for "strictly less" theorem (continued):

- *Monotonic auction*: The quantity assigned to each bidder is weakly increasing in type.
- *Value regularity*: Raising one's own type weakly increases one's ranking in values, compared to other bidders.
- *MR monotonicity*: Raising one's own type weakly increases everybody's MR.
- *High type condition*: The highest type of any agent is never a net reseller.

Theorem 6. *Consider a monotonic auction followed by strictly-individually-rational,* ***perfect*** *resale. If the ex ante probability of resale is strictly positive, then the seller's expected revenues are* ***strictly less*** *than the resale-constrained optimum.*

Get it right the first time, or it will cost you!

Application 1
Treasury Auction

In the model without resale, the revenue ranking of:

Pay-as-bid auction

Uniform-price auction

Vickrey auction

is inherently ambiguous.

Application 1
Treasury Auction

In the model with perfect resale, if each bidder's value depends exclusively on his own type, then:

Vickrey auction

unambiguously revenue-dominates:

Pay-as-bid auction

and:

Uniform-price auction.

Application 2
IPOs

Sycamore's highly anticipated initial public offering was priced at $38, but began trading at $270.875. The shares closed at $184.75, an increase of 386 percent. [T]he stock opened at 12:45 P.M. amid what one person close to the deal described as a "feeding frenzy." Within 15 minutes, the stock rose to about $200, where it remained for most of the afternoon. About 7.5 million shares were sold in the offering, or about 10 percent of the company, and 9.9 million shares traded hands yesterday. It appeared that most of the institutional investors who had been able to buy at the offering price sold quickly to those who had been shut out. The day's explosive trading could raise questions about whether the deal's underwriters left money on the table that went to the initial institutional buyers of the stock rather than to Sycamore. (*New York Times*, October 23, 1999)

Application 2
IPOs

Table 1. Recent IPO First-Day Premiums and Volume[a]

Company	Offering Price	First-Day Closing Price	Premium (First-Day Close / Offering Price)	First-Day Trading Volume / Number Shares Offered
Avis Rent-A-Car, Inc.	17	22.5	32.4%	74.8%
eBay Inc.	18	47.4	163.2%	259.9%
Guess?, Inc.	18	18.0	0.0%	53.9%
Keebler Foods Company	24	26.8	11.7%	57.9%
Pepsi Bottling Group, Inc.	23	21.7	−5.7%	61.6%
Polo Ralph Lauren Corp.	26	31.5	21.2%	67.7%
Priceline.com Inc.	16	69.0	331.3%	131.4%

It's optimal to be efficient.