Auctioning Securities

Lawrence M. Ausubel and Peter Cramton*

University of Maryland

March 1998

(first draft February 1997)

Abstract

Treasury debt and other divisible securities are traditionally sold in either a pay-your-bid (discriminatory) auction or a uniform-price auction. We compare these auction formats with a Vickrey auction and also with two ascending-bid auctions. The Vickrey auction and the alternative ascending-bid auction (Ausubel 1997) have important theoretical advantages for sellers. In a setting without private information, these auctions achieve the maximal revenue as a unique equilibrium in dominant strategies. In contrast, the pay-your-bid, uniform-price, and standard ascending-bid auction admit a multiplicity of equilibria that yield low revenues for the seller. We show how these results extend to a setting where bidders have affiliated private information. Our results question the standard ways that securities are offered to the public.

JEL No.: D44 (Auctions)
Keywords: Auctions, Multi-Unit Auctions, Security Auctions, Treasury Auctions

Send comments to:

Professors Lawrence M. Ausubel or Peter Cramton
Department of Economics
University of Maryland
College Park, MD 20742-7211

ausubel@econ.umd.edu (301) 405-3495

cramton@umd.edu (301) 405-6987

*The authors gratefully acknowledge the support of the National Science Foundation. We appreciate valuable comments from Jaime Zender and participants at numerous conferences and seminars.

Auctioning Securities

Lawrence M. Ausubel and Peter Cramton

1 Introduction

The standard analysis of the Treasury auction has contrasted two particular auction formats: the pay-your-bid auction (also known as the discriminatory auction); and the uniform-price auction (also known as the nondiscriminatory or competitive auction). The pay-your-bid auction has been the traditional method of auctioning Treasury securities; authors beginning with Milton Friedman (1960) have argued the superiority of the uniform-price auction. Today, uniform-price auctions are used for issuing two-year, five-year, and inflation-adjusted U.S. Treasury securities, while pay-your-bid auctions are used for issuing three-month, six-month, one-year, three-year, ten-year, and thirty-year Treasury securities.

The recent academic literature on the Treasury auction has generally taken three directions. First, Chari and Weber (1992) have drawn upon analogy with the theory of auctions of a single item to argue the superiority of the uniform-price auction. Second, Wilson (1979), Maxwell (1983), Back and Zender (1993), and Wang and Zender (1996) have modeled the uniform-price auction as "share auctions" and demonstrated that there exist a multiplicity of equilibria, some of which yield the seller very low revenues. Third, Malvey, Archibald and Flynn (1996) and Reinhart and Belzer (1996) have used the Treasury's recent experiment with uniform pricing to compare the pay-your-bid and uniform-price formats.

The current paper takes a substantially-different direction. We begin with the observation that there is only an imperfect analogy between the second-price auction of a single item and the uniform-price auction of multiple items. The flaw in the analogy is that, with private values, sincere bidding is a weakly-dominant strategy in the second-price auction of a single item,[1] but is not even an equilibrium strategy in the uniform-price auction of multiple items (Ausubel and Cramton 1996). However, the analogy is restored if we instead consider the multi-unit Vickrey auction (Vickrey 1961). Hence, it seems natural — rather than contrasting the pay-your-bid format with the uniform-price format — to set our sights on the Vickrey auction as a superior, sealed-bid auction for securities.

We continue with the observation that there is also only an imperfect analogy between the English auction of a single item and the standard ascending-bid auction of multiple items. Again, the flaw in the

[1] To be more precise, sincere bidding is a weakly-dominant strategy in the ascending-clock version of the English auction: the auctioneer continuously raises the price, bidders signify whether they remain in the auction, and the auction ends when only one bidder remains in the auction. Sincere bidding is *not* a weakly-dominant strategy when bidders name their bids (Avery 1998).

analogy is that, with private values, sincere bidding is a weakly-dominant strategy of the English auction of a single item, but is not an equilibrium strategy in the standard ascending-bid auction of multiple items. However, the analogy is restored if we instead consider the alternative ascending-bid auction proposed by Ausubel (1997). Whereas the *Joint Report on the Government Securities Market* (1992) advocated exploration of the standard ascending-bid auction, it seems more natural to investigate the alternative ascending-bid auction as a dynamic auction design for securities.

Once the proper analogies are developed, our paper is patterned closely after the prior literature on single-item auctions. When symmetric bidders for a single item have a pure common value and common uncertainty, the unique equilibrium in undominated strategies of the second-price auction maximizes the seller's expected revenues (subject to not using any reserve price). Similarly, here, we show that when symmetric bidders for multiple items have a pure common value and common uncertainty, the unique equilibrium in undominated strategies of the Vickrey auction maximizes the seller's expected revenues (subject to not using any reserve price). Furthermore, just as the English auction of a single item achieves the same outcome in this context as the second-price auction, the alternative ascending-bid auction achieves the same outcome in this context as the Vickrey auction.

When symmetric bidders for a single item have a pure common value, and when they receive strictly-affiliated signals of that value, Milgrom and Weber (1982) proved that the first-price auction yields less expected revenue than the second-price auction, which in turn yields less expected revenue than the English auction. By virtually the same argument, we show that all symmetric equilibria of the pay-your-bid auction for multiple items yields less expected revenue than the simple equilibrium of the Vickrey auction. Furthermore, the seller's most-preferred equilibrium of the uniform-price auction coincides with the simple equilibrium of the Vickrey auction. Finally, the simple equilibrium of the Vickrey auction in turn yields less revenue than the simple equilibrium of the alternative ascending-bid auction.

One might imagine that the distinction between uniform and Vickrey pricing is of little empirical importance in Treasury auctions, because Treasury auctions are too competitive for strategic factors to play an important role. However, the empirical reality is that Treasury auctions are dominated by a handful of large bidders. The top five primary dealers typically buy one-half of the issue in U.S. Treasury auctions (Malvey, et al. 1996). Auctions in other important settings, such as radio spectrum and electric power, also involve large bidders with significant market power.

Our paper proceeds as follows. In Section 2, we present the model and define the various auction formats studied herein. In Section 3, we obtain results for the special case when there is no private information. In Section 4, we obtain results when bidders receive affiliated private signals. We conclude in Section 5 by asking the additional question: "Why are not IPO's sold by auction?"

2 The Model and Auction Formats

2.1 The Model

We consider a homogeneous security such as a government bond, a tradeable emissions permit, or a share of a company's stock. For analytic convenience, we assume that the security is arbitrarily divisible, so that any quantity between zero and the total available supply may be purchased. Without loss of generality, we normalize the total available supply to equal one, so that any quantity $q \in [0,1]$ may be purchased.

We analyze a situation where the security has a pure common value. The random variable representing the pure common value is denoted V and the realization of the random variable is denoted v. Bidders are assumed to be risk neutral, so that the (ex post) payoff of bidder i ($i = 1,\ldots,n$) who purchases quantity q_i of the security at a unit price of p equals $(v - p)\, q_i$.

Each of the n bidders potentially receives private information about the common value. The random variable representing bidder i's signal is denoted S_i, and the realization of the random variable is denoted s_i. We assume that each S_i is real valued, with support on the interval $[\underline{s}, \overline{s}]$. Furthermore, let $S \equiv (S_1,\ldots,S_n)$ denote the vector of random variables and let $s \equiv (s_1,\ldots,s_n)$ denote the vector of realizations. Let $f(v,s)$ denote the joint probability density of the common value and the bidders' signals. We will generally make the following assumptions:

ASSUMPTION 1. The expectation of V is finite: $EV < \infty$.

ASSUMPTION 2. The random variables $V, S_1,\ldots,S_n$ are affiliated: For every $(v,s) \equiv (v,s_1,\ldots,s_n)$ and $(v',s') \equiv (v',s_1',\ldots,s_n')$, we have:

(1) $$f(v \vee v', s \vee s')\, f(v \wedge v', s \wedge s') \geq f(v,s)\, f(v',s'),$$

where $z \wedge z'$ $(z \vee z')$ denotes the component-wise maximum (minimum) of z and z'.

ASSUMPTION 3. $f(\cdot,\cdot,\ldots,\cdot)$ is symmetric in its last n arguments.

The assumption of affiliation is standard in the auctions literature, following Milgrom and Weber (1982). For readers not familiar with this notion, it may be useful to think of affiliation as non-negative correlation. When inequality (1) is satisfied with strict inequality, the condition will be referred to as strict affiliation, which may also be thought of as positive correlation.

The interpretation of affiliation is natural in the context of securities auctions. Affiliation means that, if bidder i were to infer that bidder j had received a higher signal than previously believed, then bidder i would not reduce its estimate of the common value of the security. Moreover, strict affiliation means that, if bidder i were to infer that bidder j had received a higher signal than previously believed, then bidder i would increase its estimate of the common value of the security.

Assumption 1 is required to assure that equilibrium auction prices are finite. Assumption 3 is required for the existence of symmetric equilibria to be plausible.

2.2 Sealed-Bid Auction Formats

In a multi-unit sealed-bid auction, bidders can be thought to each submit several bids, which are honored in descending order of price. Equivalently, bidders can be thought to each submit downward-sloping demand curves.

To be more precise, each bidder i submits a bid function $b_i(q):[0,1]\to[0,\infty)$, which is required to be right continuous and weakly decreasing. From $b_i(q)$, the auctioneer can construct an equivalent demand function $q_i(p)$, which specifies the quantity bidder i demands at the price p, and which is left continuous and weakly decreasing with $q_i(0) = 1$. (See also Ausubel and Cramton 1996, p. 8.) In all the sealed-bid formats we consider, the securities are assigned to the highest bids, until the supply of the security is exhausted. However, the auction formats will differ in the payment rules. The stop-out price, denoted p_0, is defined by:

(2) $$p_0 = \inf\left\{ p \mid \sum_{i=1}^{n} q_i(p) \le 1 \right\}.$$

If $\Sigma_i\, q_i(p_0) = 1$, then each bidder i receives $q_i(p_0)$. If $\Sigma_i\, q_i(p_0) > 1$, then the aggregate demand curve is flat at p_0 and some bidders' demands at p_0 must be rationed. If there is just a single bidder whose demand curve is flat at p, then this bidder's quantity is reduced by $\Sigma_i\, q_i(p_0) - 1$. If there are multiple bidders with demand curves flat at p_0, then quantity is allocated by proportionate rationing.

We are now ready to define the various sealed-bid auction formats. For expositional simplicity, we assume in the following definitions that the aggregate demand curve is not flat at p_0, so that there is no need for our notation to reflect any rationing. The traditional format used for the auctioning of Treasury securities is the pay-your-bid auction (also known as the discriminatory auction); bidders are awarded the quantity demanded at the stop-out price and pay their bids which exceed the stop-out price.

PAY-YOUR-BID AUCTION. Bidders simultaneously and independently submit (sealed) bid functions $b_i(q)$ (with equivalent demand curves of $q_i(p)$), each bidder i is assigned the quantity $q_i(p_0)$, and each bidder i is charged a payment P_i corresponding to its actual bids above the stop-out price p_0:

(3) $$P_i = \int_0^{q_i(p_0)} b_i(q)\,dq\ .$$

Drawing analogy from competitive markets, Milton Friedman (1960) proposed the uniform-price auction; bidders are awarded the quantity demanded at the stop-out price and everyone pays this market-clearing price for each unit bought.

UNIFORM-PRICE AUCTION. Bidders simultaneously and independently submit (sealed) bid functions $b_i(q)$ (with equivalent demand curves of $q_i(p)$); each bidder i is assigned the quantity $q_i(p_0)$; and each bidder i is charged a payment P_i based on the stop-out price p_0:

(4) $$P_i = q_i(p_0)\,p_0\ .$$

William Vickrey (1961) proposed what is now known as the Vickrey auction. Bidders are awarded the quantity demanded at the stop-out price and pay the opportunity cost of the award — the best rejected bids of the other bidders. The Vickrey pricing rule is famous in the private value setting, since then it is a dominant strategy for each bidder to bid its true value, which results in an efficient allocation. In a common value setting, the dominant strategy result typically is lost, but the pricing rule is still well defined.

VICKREY AUCTION. Bidders simultaneously and independently submit (sealed) bid functions $b_i(q)$ (with equivalent demand curves of $q_i(p)$), each bidder i is assigned the quantity $q_i(p_0)$, and each bidder i is charged a payment P_i corresponding to the opportunity cost of its bids

(5) $$P_i = \int_{1-q_i(p_0)}^{1} b_{-i}(q)\,dq\ ,$$

where $b_{-i}(q)$ denotes the aggregate bid function of all bidders j ($j \neq i$), i.e., the inverse of $q_{-i}(p) = \sum_{j \neq i} q_j(p)$. Equivalently, the payment rule may instead be written directly in terms of the demand curves: $P_i = q_i(p_0)\,p_0 - \int_{p_{-i}}^{p_0} [1 - q_{-i}(x)]\,dx$ where p_{-i} is the stop-out price if bidder i did not bid.

The payment rule of the Vickrey auction is easiest understood within the context of an auction for M indivisible items. There, the payment rule reduces to: bidder i pays the highest-rejected bid for the first item it wins, the second-highest rejected bid for the second item it wins, etc., where the enumeration of rejected bids for calculating bidder i's payment always excludes the bids of bidder i.

2.3 Ascending-Clock Auction Formats

In the dynamic auction formats studied here, the auctioneer can be thought to name a price p, bidders respond by each naming the quantity which they demand at that price, the auctioneer then increments the price, bidders again respond by each naming the quantity which they demand at the incremented price, and the process continues until the aggregate demand is less than or equal to the price. Any such auction format may be referred to as an *ascending-clock auction*.

To be more precise, each bidder i indicates a quantity, $q_i(p)$, demanded at price p, and the price is incremented until the final price, denoted p_0, is reached, where p_0 is defined by

$$(6) \qquad p_0 = \inf\left\{ p \mid \sum_{i=1}^{n} q_i(p) \le 1 \right\}.$$

As in the definition of the sealed-bid auction formats, if $\Sigma_i\, q_i(p_0) = 1$, then each bidder i receives $q_i(p_0)$, but if $\Sigma_i\, q_i(p_0) > 1$, then some bidders' demands at p_0 will need to be rationed.

We are now ready to define two dynamic auction formats. For expositional simplicity, we will assume in the following definitions that the aggregate demand curve is not flat at p_0, so that there is no need for our notation to reflect any rationing. The *Joint Report on the Government Securities Market* (1992) advocated exploration of the ascending-clock auction with a uniform-price rule: bidders are awarded the quantity demanded at the final price, and pay a unit price equaling the final price.

STANDARD ASCENDING-BID AUCTION. Bidders simultaneously report nonincreasing demands $q_i(p)$ according to an ascending clock until a final price of p_0 is reached, where there is no excess demand. Each bidder i is then assigned the quantity $q_i(p_0)$, and each bidder i is charged a unit price equaling the final price p_0.

Drawing analogy from the English auction of a single item, Ausubel (1997) recently proposed a new dynamic auction whose static representation is the Vickrey auction: bidders are awarded the quantity demanded at the final price, but pay according to the prices at which they “clinch” units.

ALTERNATIVE ASCENDING-BID AUCTION. Bidders simultaneously report nonincreasing demands $q_i(p)$ according to an ascending clock until a final price of p_0 is reached. At “each” price p until the final price, and for each remaining bidder i, the auctioneer calculates

$$(7) \qquad C_i(p) = \max\left\{ 0,\, A(p) - \sum_{j \ne i} q_j(p) \right\},$$

where $A(p)$ denotes the available supply remaining at price p. Bidder i is deemed to have “clinched” $C_i(p)$ units at a price of p. The available supply is then updated by subtracting out from $A(p)$ and bidder i's remaining demand is updated by subtracting out $C_i(p)$ from $q_i(p)$. Bidder i is thus awarded a quantity of $q_i(p_0)$ for a total payment of

$$(8) \qquad P_i = \int_0^{p_0} C_i(p)\, p\, dp .$$

The reader may be wondering — for completeness — if there is a dynamic auction corresponding to the pay-your-bid auction. Thus, we conclude this section by remarking that, just as the first-price auction of a single item is strategically equivalent to the descending-bid dynamic auction (known in the economics literature as the Dutch auction), it is possible to define a descending-bid auction for multiple items which is equivalent to the pay-your-bid auction. However, this auction will not be studied here.

3 Results for Bidders without Private Information

In this section, we consider a model where bidders receive no private signals. Formally, this is a special case of the model of Section 2, where for every bidder *i*, the signal S_i is independent of the pure common value *V*. Thus, bidders are symmetrically informed.

Wilson (1979) first considered this model, demonstrating the existence of equilibria of the uniform-price auction which yield the seller low revenues. Maxwell (1983) established a folk theorem for the uniform price auction: for any level of revenues below some maximum, there exists an equilibrium of the uniform-price auction yielding this level of revenues. Back and Zender (1993) argued that Wilson's equilibrium for this case is fragile, since the equilibrium quantities are deterministic, and so each bidder's bid function is only required to be optimal at a single quantity. They introduced stochastic supply (motivated by a random number of noncompetitive bids in the Treasury auction), and proved that a folk theorem for the uniform-price auction still holds. Wang and Zender (1996) further demonstrate the existence of multiple equilibria in the pay-your-bid auction.

The intuition for low-revenue equilibria in the uniform-price auction is compelling. In a uniform-price auction, the inframarginal part of the demand curve is irrelevant to the bidder. However, how by submitting a steep demand curve, the bidder increases the cost to other bidders of grabbing extra quantity. Thus, steep demand curves enforce a low price by making the marginal cost of asking for more arbitrarily large. Although the intuition is clearest in the case without uncertainty, Back and Zender (1993) show that it extends to a setting with uncertain supply.

We begin by restating the existing results on the uniform-price and pay-your-bid auctions.

PROPOSITION 3.1 (Wilson 1979, Maxwell 1983, Back and Zender 1993). For the pure common value model without private information, there exists a multiplicity of equilibria in the uniform-price auction, even if weakly-dominated strategies are eliminated and if stochastic supply is introduced. In the equilibrium with highest revenues, the seller's revenues equal *EV*.

PROPOSITION 3.2 (Wang and Zender 1996). For the pure common value model without private information, there exists a multiplicity of equilibria in the pay-your-bid auction, even if weakly-dominated strategies are eliminated and if stochastic supply is introduced. In the equilibrium with highest revenues, the seller's revenues equal *EV*.

Both the uniform-price and pay-your-bid auctions offer the possibility of raising revenues equal to the full expected value of the securities, but they also exhibit other equilibria which yield much lower revenues. Indeed, Back and Zender (1993) and Wang and Zender (1996) argue on Pareto grounds that the symmetric equilibria which minimize the seller's revenue are the most plausible.

By contrast, the Vickrey auction in this case offers a unique equilibrium, and it maximizes the seller's revenue, as we see in the following theorem:

THEOREM 3.3. For the pure common value model without private information:

(a) the Vickrey auction exhibits a unique equilibrium which survives elimination of weakly-dominated strategies;

(b) the Vickrey auction exhibits a unique symmetric equilibrium consistent with stochastic supply over the interval [0,1]; and

(c) this equilibrium revenue-dominates all equilibria of all auction formats consistent with voluntary bidder participation.

PROOF.

(a) We will show that the bid function $b_i(q) = EV$, for all $q \in (0,1]$, weakly dominates all other bid functions in the Vickrey auction. Consider any other bid function $b_i'(\cdot)$, and compare the payoffs of b_i and b_i' to bidder *i* against any aggregate bid function, $b_{-i}(\cdot)$, of the other bidders. Let q_i denote the quantity awarded to bidder *i* from b_i and b_{-i}, and let q_i' denote the quantity awarded to bidder *i* from b_i' and b_{-i}. If $q_i' < q_i$, observe from Eq. (5) that bidder *i* is charged the same price for the quantity q_i' under either bid function; however, using b_i, bidder *i* also receives quantity $q_i - q_i'$ for a unit price of at most *EV*, so b_i yields bidder *i* the same or higher payoff as b_i'. Similarly, if $q_i' > q_i$, observe from Eq. (5) that bidder *i* is charged the same price for the quantity q_i under either bid function; however, using b_i', bidder *i* also receives quantity $q_i' - q_i$ for a unit price of at least EV, so b_i again yields bidder *i* the same or higher payoff as b_i'.

(b) With stochastic supply over the interval [0,1], in any symmetric equilibrium, any point on the bid function $b_i(q)$, $q \in [0,1/n]$, may be pivotal. If $b_i(q) \neq EV$, for any $q \in [0,1/n]$, any bidder can profitably deviate by setting its bid uniformly equal to *EV*, by the same calculation as in part (a).

(c) Since the security has a pure common value, the sum of the bidders' expected values derived from the security equals *EV* in any equilibrium. Hence, in any equilibrium of any auction consistent with voluntary bidder participation, the sum of the bidders' expected payments may be at most *EV*. This maximum is attained by the Vickrey auction. ■

The Vickrey auction, once weakly-dominated strategies are eliminated, avoids the indeterminacy of the uniform-price auction. We see that the dominant strategy argument extends to the case of a divisible good. A bidder's demand curve does not affect how much the bidder pays, nor does it affect the incentives of the other bidders. The bidder simply wishes to win units whenever the price is below its value, and hence bids *EV* for all units.

Now, let us briefly consider the two ascending-clock auction formats. If bidders are provided no bid information, i.e. if they are told after each price only whether the auction has yet reached the final price, then the standard ascending-bid auction is strategically equivalent to the uniform-price auction and the alternative ascending-bid auction is strategically equivalent to the Vickrey auction. We thus have:

COROLLARY 3.4. For the pure common value model without private information:

(a) there exists a multiplicity of equilibria in the standard ascending-bid auction with no bid information — even if weakly-dominated strategies are eliminated and if stochastic supply is introduced — and the seller's revenues are bounded above by EV;

(b) the alternative ascending-bid auction with no bid information exhibits a unique equilibrium which survives elimination of weakly-dominated strategies, as well as a unique symmetric equilibrium consistent with stochastic supply over the interval [0,1]; and

(c) this equilibrium revenue-dominates all equilibria of all auction formats consistent with voluntary bidder participation.

4 Results for Bidders with Affiliated Private Signals

In this section, we consider the full model of Section 2. Every bidder i ($i = 1,\ldots,n$) receives a signal S_i, and the random variables $V, S_1,\ldots,S_n$ are affiliated. Thus, the bidders possess private information, and bidder *j*'s private information is relevant to the assessment of bidder *i*'s value. The model is a generalization of the frameworks considered by Wilson (1979) and Back and Zender (1993).

Most or all of the auction formats exhibit a multiplicity of equilibria for this model. However, there exist bounds on the set of equilibria provided by the theory of auctions of single items. In particular, the revenue of all equilibria of the pay-your-bid auction is bounded above by the revenue of the first-price auction for a single item; the revenue of all equilibria of the uniform-price and Vickrey auctions are bounded above by the revenue of the second-price auction for a single item; and the revenue of all equilibria of the standard ascending-bid and alternative ascending-bid auctions are bounded above by the revenue of the English auction for a single item. Under the assumption of affiliation, these respective upper bounds may be ranked. In addition, as in the case of no private bidder information, the Vickrey auction reduces the multiplicity of equilibria of the uniform-price auction, and the alternative ascending-bid auction reduces the multiplicity of equilibria of the standard ascending-bid auction.

In order to state our results, it is helpful to define the following standard notation. Let the first-order statistic, $S_{(1)}$, of $S_1,\ldots,S_n$ be defined by: $S_{(1)} = \max\{S_1,\ldots,S_n\}$. Let the second-order statistic, $S_{(2)}$, be defined by: $S_{(2)} = \max\{\{S_1,\ldots,S_n\}\backslash\{S_{(1)}\}\}$.

PROPOSITION 4.1. For the pure common value model with affiliated private signals, the following is a symmetric equilibrium of the pay-your-bid auction: Each bidder i submits a flat bid function, $b_i(q;s_i) = b_i(s_i)$, for $q \in [0,1]$, where $b_i(s_i)$ is the symmetric equilibrium of the first-price auction of a single item. [Conjecture: Moreover, the expected revenues of this equilibrium constitute an upper bound on the revenues of all symmetric equilibria of the pay-your-bid auction.]

PROPOSITION 4.2. For the pure common value model with affiliated private signals, the following is a symmetric equilibrium of the uniform-price auction and the Vickrey auction: Each bidder i submits a flat bid function, $b_i(q;s_i) = E\{V \mid S_{(1)} = s_i, S_{(2)} = s_i\}$, for $q \in [0,1]$, which corresponds to the symmetric equilibrium of the second-price auction of a single item. [Conjecture: Moreover, the expected revenues of this equilibrium constitute an upper bound on the revenues of all symmetric equilibria of the uniform-price auction and Vickrey auctions.]

PROPOSITION 4.3. For the pure common value model with affiliated private signals, the following is a symmetric equilibrium of the standard ascending-bid auction and the alternative ascending-bid auction: Each bidder i demands a quantity $q(p;\, s_i) = 1$, until the same circumstances occur under which it drops out of the corresponding English auction of a single item, and then it demands a quantity $q(p;\, s_i) = 0$. [Conjecture: Moreover, the expected revenues of this equilibrium constitute an upper bound on the revenues of all symmetric equilibria of the standard ascending-bid and alternative ascending-bid auctions.]

The equilibria in these propositions are simple in that each bidder is submitting a flat bid function for the entire quantity available. As a result, a bidder wins either all or none of the issue. Moreover, the residual supply curve is flat for all quantities between 0 and 1. The existence of these simple equilibria depends on the assumption that each bidder can purchase the entire quantity available. If, instead, bidder i has a capacity $\lambda_i < 1$, then flat bid functions are not possible in equilibrium, except when all bidders have the same capacity ($\lambda_i = \lambda$ for all i) and $1/\lambda$ is an integer. In this special case, if all bidders submit flat bid functions, each bidder will face a flat residual supply curve for its entire capacity.

THEOREM 4.4. For the pure common value model with (strictly) affiliated private signals, the expected revenue of the equilibrium of Proposition 4.1 is (strictly) less than the expected revenue of the equilibrium of Proposition 4.2, which in turn is (strictly) less than the expected revenue of the equilibrium of Proposition 4.3.

PROOF. The first inequality follows directly from the same argument as Milgrom and Weber (1982, Theorem 15). The second inequality follows directly from the same argument as Milgrom and Weber (1982, Theorem 11). ■

CONJECTURE 4.5. For the pure common value model with affiliated private signals:

(a) the Vickrey auction eliminates some of the indeterminacy of the uniform-price auction (at the lower end of revenues); and

(b) the alternative ascending-bid auction eliminates some of the indeterminacy of the standard ascending-bid auction (at the lower end of revenues).

PROOF.

(a) It is a weakly-dominated strategy in the Vickrey auction for any bidder to bid less than $b_i(q;\underline{s}) = E\{V \mid S_{(1)} = \underline{s}, S_{(2)} = \underline{s}\}$, for $q \in [0,1]$. Consequently, the interim utility of a bidder receiving a signal of $\underline{s}$ (the lowest-possible signal) is forced to equal zero, in any equilibrium surviving elimination of weakly-dominated strategies. This bound is not present in equilibria of the uniform-price auction (Wilson 1979, Back and Zender 1993).

(b) Follows by similar reasoning. ■

We further conjecture that stronger results reducing the multiplicity of equilibria in the Vickrey auction and alternative ascending-bid auction may be obtained. Indeed, there is some possibility that the equilibrium of the Vickrey auction displayed in Proposition 4.2 may be the unique symmetric equilibrium surviving elimination of weakly-dominated strategies.

5 Conclusion

In any securities auction in which bidders possess private information, one should expect there to be “underpricing,” in the sense that the seller's expected revenues should be less than the expected value of the securities being auctioned. This is a direct consequence of the bidders obtaining informational rents from their private information. The seller's interest, then, should be to adopt an auction for securities that limits the extent of underpricing.

We have seen in this paper that the Vickrey auction and the alternative ascending-bid auction have theoretical advantages over the pay-your-bid auction and uniform-price auction, which are currently used to sell government securities. When bidders do not have private information, underpricing is eliminated by a Vickrey or alternative ascending-bid auction. When bidders have private information, then the extent of underpricing is reduced with Vickrey and alternative ascending-bid formats.

Given the empirical fact that underpricing in the U.S. Treasury auctions is small, the potential gains from a change in the auction format are modest. Still, a reduction in borrowing costs of even one basis point on a gross public debt of more than $5 trillion (of which approximately $3.4 trillion is in private hands) is not something to entirely disregard.

However, in the case of initial public offerings of corporate stock, the magnitude of underpricing under current American practice appears to be vastly larger than necessary. The substantial underpricing is indicative of a badly-performing mechanism for selling new issues. Hence, we conclude this paper by raising the question: “Why are IPO's not done instead by an efficient auction?”

As an example, let us consider an IPO such as Netscape, where it is known in advance that the IPO will be vastly oversubscribed at the offering price and that there will be a considerable degree of underpricing. Would it not have been in the company's interest to sell its stock through one of the methods described in this paper? While relatively obscure new issues might benefit from the certification and marketing afforded by the underwriting process, it is hard to see how an already known business such as Netscape would not raise higher revenues from using an auction.

It is our opinion that, while some explanations can be put forth why auctions are not today used as the preferred method of offering corporate securities, these reasons rely largely on institutional inertia. Indeed, the incumbent corporate underwriters possess a strong profit motive in discouraging the advent of auctions, as they are the beneficiaries of today's substantial underpricing. And whereas auction formats with ascending bids — which are believed to perform the best — were technologically infeasible only a few years ago, they are eminently workable today with the rise of the Internet and other computer communications advances.

References

Ausubel, Lawrence M. (1997), “An Efficient Ascending-Bid Auction for Multiple Objects,” Working Paper 97-06, University of Maryland.

Ausubel, Lawrence M. and Peter Cramton (1996), “Demand Reduction and Inefficiency in Multi-Unit Auctions,” Working Paper No. 96-07, University of Maryland.

Avery, Christopher (1998), “Strategic Jump Bidding in an English Auction,” *Review of Economic Studies*, forthcoming.

Back, Kerry and Jaime F. Zender (1993), “Auctions of Divisible Goods: On the Rationale for the Treasury Experiment,” *Review of Financial Studies*, 6, 733-764.

Bikhchandani, Sushil (1996), “Auctions of Heterogeneous Objects,” Working Paper, UCLA.

Bolle, Friedel (1995), “Necessary Conditions for Efficient Multiple-Bid Auctions,” Working Paper, Europa-Universitat Viadrina.

Bulow, Jeremy and Paul Klemperer (1996), “Auctions vs. Negotiations,” *American Economic Review*, 86, 180-194.

Chari, V. V. and Robert J. Weber (1992), “How the U.S. Treasury Should Auction Its Debt,” *Quarterly Review*, Federal Reserve Bank of Minneapolis, 3-12.

Engelbrecht-Wiggans, Richard (1988), “Revenue Equivalence in Multi-object Auctions,” *Economic Letters*, 26, 15-19.

Engelbrecht-Wiggans, Richard and Charles M. Kahn (1995), “Multi-Unit Auctions with Uniform Prices,” Working Paper, University of Illinois.

Friedman, Milton (1960), *A Program for Monetary Stability*, New York, NY: Fordham University Press.

Harstad, Ronald M. (1990), “Alternative Common-Value Auction Procedures: Revenue Comparisons with Free Entry,” *Journal of Political Economy*, 98, 421-429.

Katzman, Brett Eric (1995), “Multi-Unit Auctions with Incomplete Information,” Working Paper, Duke University.

Levin, Dan and James L. Smith (1994), “Equilibrium in Auctions with Entry,” *American Economic Review*, 84, 585-599.

Levin, Dan and James L. Smith (1996), “Optimal Reservation Prices in Auctions,” *Economic Journal*, 106, forthcoming.

Malvey, Paul F., Christine M. Archibald, and Sean T. Flynn (1996), “Uniform-Price Auctions: Evaluation of the Treasury Experience,” Working Paper, U.S. Treasury.

Maskin, Eric S. and John G. Riley (1989), “Optimal Multi-unit Auctions,” in Frank Hahn (ed) *The Economics of Missing Markets, Information, and Games*, Oxford: Oxford University Press, 312-35.

Maxwell, Christopher (1983), “Auctioning Divisible Commodities: A Study of Price Determination,” Doctoral Dissertation, Harvard University.

Milgrom, Paul and Robert J. Weber (1982), “A Theory of Auctions and Competitive Bidding,” *Econometrica*, 50, 1089-1122.

Myerson, Roger B. (1981), “Optimal Auction Design,” *Mathematics of Operations Research*, 6, 58-73.

Noussair, Charles (1995), “Equilibria in a Multi-Object Uniform Price Sealed Bid Auction with Multi-Unit Demands,” *Economic Theory*, 5, 337-351.

Reinhart, Vincent and Gregory Belzer (1996), “Some Evidence on Bid Shading and the Use of Information in the U.S. Treasury's Auction Experiment,” Working Paper, Board of Governors of the Federal Reserve System.

Rothkopf, Michael H., Thomas J. Teisberg, and Edward P. Kahn (1990), “Why Are Vickrey Auctions Rare?” *Journal of Political Economy*, 98, 94-109.

Smith, Vernon L. (1967), “Experimental Studies of Discrimination versus Competition in Sealed-Bid Auction Markets,” *Journal of Business*, 40, 58-84.

Smith, Vernon L. (1982), “Microeconomic Systems as an Experimental Science,” *American Economic Review*, 72, 923-955.

Tenorio, Rafael (1996), “On Strategic Quantity Bidding in Multiple Unit Auctions,” Working Paper, Notre Dame University.

Department of the Treasury, Securities and Exchange Commission, and Board of Governors of the Federal Reserve System (1992), *Joint Report on the Government Securities Market*, U.S. Government Printing Office, January.

Vickrey, William (1961), "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16, 8-37.

Wang, James J.D. and Jaime F. Zender (1996), "Auctioning Divisible Goods," Working Paper, University of Utah.

Weber, Robert J. (1983), "Multiple-Object Auctions," in Richard Engelbrecht-Wiggans, Martin Shubik, and Robert M. Stark (eds) *Auctions, Bidding, and Contracting: Uses and Theory*, New York: New York University Press, 165-191.

Wilson, Robert (1979), "Auction of Shares," *Quarterly Journal of Economics*, 94, 675-689.