Applicant Auctions for Top-Level Domains

Using auctions to efficiently resolve conflicts among applicants

Peter Cramton, University of Maryland
Ulrich Gall, Stanford University
Pat Sujarittanonta, Cramton Associates
Robert Wilson, Stanford University

www.ApplicantAuction.com
@ApplicantAuc
28 March 2013
The top-level domains (items)
The applicants (bidders)
<table>
<thead>
<tr>
<th>Summary numbers</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total applications</td>
<td>1930</td>
</tr>
<tr>
<td>Contested applications</td>
<td>755</td>
</tr>
<tr>
<td>Contested domains</td>
<td>232</td>
</tr>
<tr>
<td>Applicants</td>
<td>444</td>
</tr>
<tr>
<td>Applicants holding a contested application</td>
<td>145</td>
</tr>
</tbody>
</table>
Applicant Auction Plan

Auction design (August to December)

• Development
• Testing
• Education

First auction consultation (December to April)

• Conference and mock auction (18 Dec, Santa Monica)
• Consultation

First Applicant Auction (late April)

• First commitment
• Mock auction
• Live auction
• Settlement

Second Applicant Auction (July)

• Second commitment
• Mock auction
• Live auction
• Settlement

Third Applicant Auction (September)

• Third commitment
• Mock auction
• Live auction
• Settlement
Key benefits of applicant auctions

• *Avoids delay and value loss from ICANN Last Resort Auction*
• Maximize value of domains (puts them to their best use)
• Rapidly resolve contention leading to faster ICANN assignment
• Allow the applicants retain benefits of resolution, rather than sharing benefits with ICANN
• Lower price paid by buyer (applicant with highest bid)
• Compensate sellers (applicants with lower bids) with a share of buyer’s payment
Auction objectives

- **Efficiency.** Auction maximizes applicant value
- **Fairness.** Auction is fair. Each applicant is treated same way; no applicant is favored in any way
- **Transparency.** Auction has clear and unambiguous rules that determine the allocation and associated payments in a unique way based on the bids received
- **Simplicity.** Auction is as simple as possible to encourage broad participation and understanding
The power of mechanism design:

*Equal shares supports efficiency and fairness objectives*

• Assume:
  – Each bidder’s value is drawn independently from the uniform distribution on $[0, v_{\text{max}}]$  
  – Each bidder seeks to maximize dollar profit  
  – High bidder wins; non-high bidders share winner’s payment equally  
  – Consider 1\(^{st}\)-price and 2\(^{nd}\)-price pricing rules

• **Proposition.** *There is a unique equilibrium, the outcome is ex post efficient, and each bidder’s profit is invariant to the pricing rule (revenue equivalence).*

• **Proof.** Direct calculation results in a unique increasing equilibrium. Efficiency then is obvious. Revenue equivalence holds because the interim payment of the lowest-value bidder is invariant to the pricing rule.
But revenue equivalence does not hold for all distributions

• Assume:
  – Each bidder’s value is drawn independently from the same distribution $F$ with positive density $f$ on $[0, v_{\text{max}}]$
  – Each bidder seeks to maximize dollar profit
  – High bidder wins; non-high bidders share winner’s payment equally
  – Consider any pricing rule (e.g. 1st price, 2nd price, ...) that results in an increasing equilibrium bid function

• **Theorem.** *The outcome is ex post efficient. However, a bidder’s expected profit depends on the pricing rule (revenue equivalence fails).*

• **Proof.** Efficiency is obvious. Revenue equivalence does not hold because the interim payment of the lowest-value bidder is non-zero and depends on the pricing rule.
Counter example of revenue equivalence

- Consider an auction with three bidders whose values are distributed according to $F(x) = x^2$
- As shown, expected payments of a bidder with zero value differ in first- and second-price auctions

Expected payment; 1st price blue, 2nd price purple

Low-value bidders prefer 1st price sealed-bid in this example
Prototype auction designs

• Sequential first-price sealed-bid auction
• Simultaneous ascending clock auction

*Both approaches have proven successful when auctioning many related items*
Addressing the holdout problem

• Applicant must make a binding commitment to participate in Applicant Auction by commitment date
  – Applicant agrees to participate in auction for all of the domains it has applied for
  – For domains lacking unanimous participation, applicant agrees to wait until the ICANN Last Resort Auction to resolve string contention

• This commitment removes “holding out and negotiating with other applicants” as a viable alternative

• All should participate since the Applicant Auction dominates the ICANN auction for all applicants
### Small guys need big guys

<table>
<thead>
<tr>
<th>Top-N</th>
<th>Top-Nth</th>
<th>Number of domains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top-1</td>
<td>Donuts</td>
<td>100.0%</td>
</tr>
<tr>
<td>Top-2</td>
<td>Minds+Machines</td>
<td>0.6%</td>
</tr>
<tr>
<td>Top-3</td>
<td>Google</td>
<td>2.3%</td>
</tr>
<tr>
<td>Top-4</td>
<td>Famous Four</td>
<td>8.6%</td>
</tr>
<tr>
<td>Top-5</td>
<td>Uniregistry</td>
<td>15.9%</td>
</tr>
<tr>
<td>Top-6</td>
<td>Alfias</td>
<td>19.0%</td>
</tr>
<tr>
<td>Top-7</td>
<td>Amazon</td>
<td>23.3%</td>
</tr>
<tr>
<td>Top-8</td>
<td>Radix</td>
<td>26.3%</td>
</tr>
<tr>
<td>Top-9</td>
<td>Fairwinds</td>
<td>28.8%</td>
</tr>
<tr>
<td>Top-10</td>
<td>Nu Dot</td>
<td>28.9%</td>
</tr>
<tr>
<td>Top-11</td>
<td>United TLD</td>
<td>31.0%</td>
</tr>
<tr>
<td>Top-12</td>
<td>Top Level Design</td>
<td>31.5%</td>
</tr>
<tr>
<td>Top-13</td>
<td>Merchant Law</td>
<td>31.9%</td>
</tr>
<tr>
<td>Top-14</td>
<td>Dish</td>
<td>34.5%</td>
</tr>
<tr>
<td>Top-15</td>
<td>TLD Asia</td>
<td>36.2%</td>
</tr>
<tr>
<td>Top-16</td>
<td>Registry</td>
<td>37.5%</td>
</tr>
<tr>
<td>Top-17</td>
<td>Wolfe</td>
<td>39.2%</td>
</tr>
<tr>
<td>Top-18</td>
<td>Dot Registry</td>
<td>40.1%</td>
</tr>
<tr>
<td>Top-19</td>
<td>CSC Info</td>
<td>40.5%</td>
</tr>
<tr>
<td>Top-20</td>
<td>Secondgen</td>
<td>41.8%</td>
</tr>
</tbody>
</table>

### Big guys need small guys

<table>
<thead>
<tr>
<th>Top-N</th>
<th>Top-Nth</th>
<th>Number of domains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top-1</td>
<td>Donuts</td>
<td>30.6%</td>
</tr>
<tr>
<td>Top-2</td>
<td>Minds+Machines</td>
<td>25.9%</td>
</tr>
<tr>
<td>Top-3</td>
<td>Google</td>
<td>15.9%</td>
</tr>
<tr>
<td>Top-4</td>
<td>Famous Four</td>
<td>13.8%</td>
</tr>
<tr>
<td>Top-5</td>
<td>Uniregistry</td>
<td>12.5%</td>
</tr>
<tr>
<td>Top-6</td>
<td>Alfias</td>
<td>10.8%</td>
</tr>
<tr>
<td>Top-7</td>
<td>Amazon</td>
<td>8.6%</td>
</tr>
<tr>
<td>Top-8</td>
<td>Radix</td>
<td>7.3%</td>
</tr>
<tr>
<td>Top-9</td>
<td>Fairwinds</td>
<td>5.6%</td>
</tr>
<tr>
<td>Top-10</td>
<td>Nu Dot</td>
<td>5.6%</td>
</tr>
<tr>
<td>Top-11</td>
<td>United TLD</td>
<td>4.7%</td>
</tr>
<tr>
<td>Top-12</td>
<td>Top Level Design</td>
<td>4.7%</td>
</tr>
<tr>
<td>Top-13</td>
<td>Merchant Law</td>
<td>4.7%</td>
</tr>
<tr>
<td>Top-14</td>
<td>Dish</td>
<td>4.7%</td>
</tr>
<tr>
<td>Top-15</td>
<td>TLD Asia</td>
<td>4.3%</td>
</tr>
<tr>
<td>Top-16</td>
<td>Registry</td>
<td>4.3%</td>
</tr>
<tr>
<td>Top-17</td>
<td>Wolfe</td>
<td>4.3%</td>
</tr>
<tr>
<td>Top-18</td>
<td>Dot Registry</td>
<td>4.3%</td>
</tr>
<tr>
<td>Top-19</td>
<td>CSC Info</td>
<td>3.4%</td>
</tr>
<tr>
<td>Top-20</td>
<td>Secondgen</td>
<td>3.4%</td>
</tr>
</tbody>
</table>
Contracts

ICANN

Neutral

Market facilitator
Cramton Associates

Applicant 1
Donuts

Applicant 2
Amazon

Applicant 3
Google

...
Deposit

• A 20% deposit is required to assure that bids are binding commitments
• Bids may not exceed five times current deposit deposit
• Deposit may increase during auction
  – As a result of selling domain rights (real-time credits to escrow account)
  – As a result of deposit top-ups (credited at end of business day)
• Deposit is held in escrow account at major international bank (Citibank)
Settlement

• Within 8 business-days of auction end, settlement is executed by the settlement agent, a major international law firm working with the major international bank

• At no time does the market facilitator have access or take title to deposits, settlement amounts, or domain rights
Experimental testing
87 items (generic top-level domains)
size indicates number of applicants
16 bidders (Applicants)
size indicates number of applications
Treatments: $2 \times 2$ experimental design

• 2 auction formats
  – Sequential first-price sealed-bid
  – Simultaneous ascending clock (second price)

• 2 value distributions (independent private value)
  – Symmetric (uniform from 0 to $5000k$)
    • 16 bidders, mean value = $2500k$
  – Asymmetric (triangle distribution from 0 to $5000k$)
    • 3 large strong bidders, mean = $3750k$
    • 13 smaller weak bidders, mean = $1250k$
Applicant Auction
Simultaneous Ascending Clock Auction - October 2012

Bidding for Amazon - Round 2

Your Bid must be at least the Minimum Price to Bid. In order to Continue, your Bid must be greater than or equal to the Minimum Price to Continue.

In the next Round, you will no longer be able to bid on those Domains that are marked "Exit".

Bid Status:  
Current Limit: $42,250k  
Total Commitment: $8,220k

[Download Bids File]  
You may submit a new Bids File

<table>
<thead>
<tr>
<th>gTLD</th>
<th>Still Bidding</th>
<th>Minimum Price to Continue</th>
<th>Bid</th>
<th>Your Private Value</th>
<th>Bid Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>.group</td>
<td>3</td>
<td>$1,820k</td>
<td>$2,584k</td>
<td>$2,173k</td>
<td>$2,260k</td>
</tr>
<tr>
<td>.mobile</td>
<td>3</td>
<td>$1,470k</td>
<td>$2,087k</td>
<td>$1,668k</td>
<td>$1,670k</td>
</tr>
<tr>
<td>.save</td>
<td>2</td>
<td>$1,020k</td>
<td>$1,448k</td>
<td>$1,193k</td>
<td>$1,080k</td>
</tr>
<tr>
<td>.dev</td>
<td>2</td>
<td>$1,020k</td>
<td>$1,448k</td>
<td>$1,126k</td>
<td>$880k</td>
</tr>
<tr>
<td>.you</td>
<td>2</td>
<td>$1,020k</td>
<td>$1,448k</td>
<td>$2,080k</td>
<td>$3,680k</td>
</tr>
</tbody>
</table>

Note:
- 'Still Bidding' is the number of Bidders with Bids of at least the Minimum Price to Bid of this Round at its start time.
- **Continue**: You will be able to bid for this Domain next Round.
- **Exit**: You will not be able to bid for this Domain next Round.
Experimental results
Clearing round and prices

In sequential, by construction, about the same number clear in each round.

In simultaneous, strong tendency for highest value domains to clear last, allowing better budget management.
Efficiency: ratio of realized to potential value

Both auction formats are highly efficient

<table>
<thead>
<tr>
<th></th>
<th>Sequential</th>
<th>Symmetric</th>
<th>Simultaneous</th>
<th>Asymmetric</th>
<th>Simultaneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value-a</td>
<td>98.1%</td>
<td>99.1%</td>
<td>97.0%</td>
<td>96.3%</td>
<td>97.1%</td>
</tr>
<tr>
<td>Value-b</td>
<td>99.0%</td>
<td>99.1%</td>
<td>98.5%</td>
<td>97.7%</td>
<td>98.0%</td>
</tr>
<tr>
<td>Value-a</td>
<td>98.4%</td>
<td>98.5%</td>
<td>97.6%</td>
<td>98.2%</td>
<td>98.6%</td>
</tr>
<tr>
<td>Value-b</td>
<td>98.5%</td>
<td>98.5%</td>
<td>98.5%</td>
<td>98.0%</td>
<td>98.3%</td>
</tr>
</tbody>
</table>
Deviation in bids from theory

In sequential, bidders tend to overbid

In simultaneous, bidders tend to underbid
Actual and equilibrium bids

In simultaneous, bidders tend to underbid in both cases.

Black: Actual = Equilibrium
Blue: Trend of actual with ±5% confidence band

In sequential, bidders tend to overbid in symmetric, but not asymmetric case.

In simultaneous, bidders tend to underbid in both cases.
Human and equilibrium bid functions (symmetric)

In sequential, bidders tend to overbid

Trend with ±5% confidence band
Equilibrium bid

In simultaneous, bidders tend to underbid

Trend with ±5% confidence band
Equilibrium bid
Conclusion

• Both auction formats perform well
  – About 98% of potential value is realized
• Preference for simultaneous ascending clock
  – Better price discovery
  – Better deposit management
  – Reduced tendency to overbid
  – More consistent with ICANN Last Resort Auction
Limitations of analysis

• Actual auction setting will have more uncertainty than assumed here
  – Value distributions will not be commonly known
  – Values will be positively correlated, not independent
  – Some bidders may be less sophisticated than others

• Uncertainty will introduce guesswork, which likely will limit efficiency

• However, since ascending auctions outperform first-price sealed-bid auctions in settings with greater uncertainty and value correlation, these complications seem to reinforce our conclusion: the simultaneous ascending format most likely is best
Appendix

Experimental instructions and examples from theory
Simultaneous ascending clock
Auction rules

Simultaneous ascending clock

- All 87 domains will be sold simultaneously in multiple rounds. In each round, for each domain, the number of active bidders is announced together with two prices: (i) the *minimum price to bid*, and (ii) the *minimum price to continue*. The *minimum price to bid* is where the auction has reached at the end of the last round (or $0 in the first round). You are already committed to a bid of at least this amount, which is why this is the lowest bid you may place. The *minimum price to continue* is the smallest bid that you may place in the current round in order to be given the opportunity to bid in the next round. Thus, for each domain of interest, the submitted bid indicates your decision to either exit in the current round with a bid that is between the *minimum price to bid* and the *minimum price to continue*, or continue with a bid that is at or above the *minimum bid to continue*, in which case you will be given the opportunity to continue bidding on the domain in the next round. In other words you may:
  - *Exit* from a domain by choosing a bid that is less than the announced *minimum price to continue* for that round. A bidder cannot bid for a domain for which she has submitted an exit bid.
  - *You may continue* to bid on a domain of interest by choosing a bid that is greater than or equal to the announced *minimum price to continue* for that round.
Symmetric values

• *Symmetric Values*: Each bidder’s value for each domain is randomly and independently drawn from a *uniform distribution* on the interval \([0, 5000]\), rounded to the nearest integer. These values are private—each bidder will know only her own value.
Profits

• Profit from domain won:

\[ \text{Profit}_{\text{won}} = \text{value} - \text{price} \]

• Profit from domain lost, where \( n \) is the initial number of bidders for the domain:

\[ \text{Profit}_{\text{lost}} = \frac{\text{winner’s payment}}{n - 1} \]
Profits (examples)

• Suppose that your valuation for the domain is 4,500 and you win it at a price of 4,000. Then your profit from this domain is equal to 4,500 – 4,000 = 500 ED.

• Suppose that you lose the domain, the initial number of bidders for that domain is 5, and the winner pays 4,000. Then your profit from this domain is equal to 4,000 / 4 = 1,000 ED.
Deposit

- Each bidder has an initial deposit. The size of the deposit determines the maximum bidding commitment the bidder can make. The total of active bids and winning payments cannot exceed five times the current deposit. As domains are sold, the payment received by the loser is added to the deposit amount. Also for domains that have not yet sold but for which the bidder has exited, the bidder’s deposit is credited with the minimum payment that the bidder may receive once the domain is sold—this is the minimum price to bid in the current round.

- The auction system will prevent a bidder from placing bids on a collection of domains that would cause the bidder’s total commitment to exceed five times the bidder’s current deposit.
Bidding strategy

*Symmetric second-price auction*

- The simultaneous ascending clock auction allows the bidders to adopt complex bidding strategy. Below are some results from auction theory about *single item auctions* that may be relevant when devising your bidding strategy.

- Before stating the results, here is some notation. There are $n$ bidders with bidder $i$ assigning a value of $V_i$ to the object. Each $V_i$ is drawn independently on the interval $[0, \bar{v}]$ according to the cumulative distribution function $F_i$ with a positive density $f_i$. 
Bidding strategy

*Symmetric second-price auction*

- Recall that in the standard private-value setting where winning payments are retained by the auctioneer, the second-price and ascending clock auctions both have the same dominant strategy equilibrium: bid (up to) your private value, or \( b(v) = v \).

- Bidder incentives change in our setting where the winner’s payment is shared equally among the losers. Notice that losing is made more attractive in this case, relative to the standard auction—the loser receives a share of the winner’s payment, rather than 0.
Bidding strategy

Symmetric second-price auction

• With symmetric bidders with values independently drawn from the uniform distribution, there is a unique symmetric equilibrium for the second-price domain auction. It is

\[ b(v) = \bar{v} \frac{(n - 1) \left( n \frac{v}{\bar{v}} + 1 \right)}{n(n + 1)}. \]
Asymmetric values

- **Asymmetric Values**: Each bidder’s value for each domain is randomly and independently drawn from a *triangle distribution* on the interval \([0, 5000]\), rounded to the nearest integer. These values are private—each bidder will know only her own value. Two types of triangle distributions are used depending on whether the bidder is *strong* or *weak*. There are three strong bidders: Donuts, Google and Amazon. The rest of the bidders are weak.
Asymmetric values

- The value of a strong bidder for each domain, $x$, is randomly and independently drawn from a distribution with density $f_s(x) = 2x$, and cumulative $F_s(x) = x^2$ on the interval $[0, 5000]$, rounded to the nearest integer. The mean value then is 3750 thousand dollars.

- The value of a weak bidder for each domain, $x$, is randomly drawn from a distribution with density $f_w(x) = 2 - 2x$, and cumulative $F_w(x) = 1 - (1 - x)^2$ on the interval $[0, 5000]$, rounded to the nearest integer. The mean value then is 1250 thousand dollars.
Bidding strategy (triangle density)

**Symmetric second-price auction**

- We can also calculate the unique symmetric equilibrium when there are two bidders and each bidder’s value is independently drawn from a triangle distribution.

- With two strong bidders, the symmetric equilibrium bid function is

  \[
  b_{2\text{strong}}(v) = \bar{v} \frac{4 + 2 \frac{v}{\bar{v}} (4 + 3 \frac{v}{\bar{v}} (2 + \frac{v}{\bar{v}}))}{15(1 + \frac{v}{\bar{v}})^2}.
  \]

- With two weak bidders, the symmetric equilibrium bid function is

  \[
  b_{2\text{weak}}(v) = \frac{\bar{v}}{10} (1 + 4 \frac{v}{\bar{v}}).
  \]
Bidding strategy

Asymmetric second-price auction

• When the bidders’ values are drawn from different distributions then numerical methods must be used to compute the equilibrium. As an example, we present the case with one strong bidder and one weak bidder in the figure below. Notice that the weak bidder bids more aggressively than the strong bidder to compensate for the weakness; similarly the strong bidder bids less aggressively than the weak bidder in recognition of her relative strength.
One weak and one strong

*Asymmetric second-price auction*
Sequential first-price sealed-bid
Auction rules

**Sequential first-price sealed-bid**

- All 87 domains will be sold in a sequence of first-price sealed-bid rounds. In each round, a small batch of domains will be auctioned simultaneously using the first-price sealed-bid format: for each domain, the high bidder wins and pays her bid. The winner’s payment is split equally among the losing bidders. Ties are broken randomly.
- The batching of domains, as well as the auction schedule for each round will be announced before the first round takes place.
- You will be able to make bids on each of the domains you applied for. At the time you place your bid you will know the set of domains you applied for (and therefore can bid on) and the set of domains each of the other bidders applied for. Thus, you will know both the number of bidders and the other companies that applied for each domain. If you fail to place a bid in the time available—either before or during the round in which the particular domain is auctioned—a bid of zero is assumed.
- After a round has ended, the winning bid amount will be disclosed, but not the identity of the winner. The winner’s deposit will be debited by one-fifth of the winning bid amount; each loser’s deposit will be increased by five times the winning bid amount.
Symmetric values

- **Symmetric Values**: Each bidder’s value for each domain is randomly and independently drawn from a *uniform distribution* on the interval $[0, 5000]$, rounded to the nearest integer. These values are private—each bidder will know only her own value.
Profits

• Profit from domain won:
  \[ \text{Profit}_{\text{won}} = \text{value} - \text{price} \]

• Profit from domain lost, where \( n \) is the initial number of bidders for the domain:
  \[ \text{Profit}_{\text{lost}} = \frac{\text{winner’s payment}}{n - 1} \]
Profits (examples)

• Suppose that your valuation for the domain is 4,500 and you win it at a price of 4,000. Then your profit from this domain is equal to 4,500 – 4,000 = 500 ED.

• Suppose that you lose the domain, the initial number of bidders for that domain is 5, and the winner pays 4,000. Then your profit from this domain is equal to 4,000 / 4 = 1,000 ED.
Deposit

• Each bidder has an initial deposit. The size of the deposit determines the maximum bidding commitment the bidder can make. The total of active bids and winning payments cannot exceed five times the current deposit. As domains are sold, the payment received by the loser is added to the deposit amount.

• The auction system will prevent a bidder from placing bids on a collection of domains that would cause the bidder’s total commitment to exceed five times the bidder’s current deposit.
Bidding strategy

*Symmetric first-price auction*

• The simultaneous ascending clock auction allows the bidders to adopt complex bidding strategy. Below are some results from auction theory about single item auctions that may be relevant when devising your bidding strategy.

• Before stating the results, here is some notation. There are $n$ bidders with bidder $i$ assigning a value of $V_i$ to the object. Each $V_i$ is drawn independently on the interval $[0, \bar{v}]$ according to the cumulative distribution function $F_i$ with a positive density $f_i$. 

Bidding strategy

**Symmetric first-price auction**

• Recall that in the standard private-value setting where winning payments are retained by the auctioneer, the first-price sealed-bid auction has a unique symmetric equilibrium: bid

\[ b(v) = \frac{n - 1}{n} v. \]

• Bidder incentives change in our setting where the winner’s payment is shared equally among the losers. Notice that losing is made more attractive in this case, relative to the standard auction—the loser receives a share of the winner’s payment, rather than 0.
Bidding strategy

Symmetric first-price auction

• With symmetric bidders with values independently drawn from the uniform distribution, there is a unique symmetric equilibrium for the first-price domain auction. It is

\[ b(v) = \frac{n - 1}{n + 1} v. \]
Asymmetric values

• *Asymmetric Values:* Each bidder’s value for each domain is randomly and independently drawn from a *triangle distribution* on the interval \([0, 5000]\), rounded to the nearest integer. These values are private—each bidder will know only her own value. Two types of triangle distributions are used depending on whether the bidder is *strong* or *weak*. There are three strong bidders: Donuts, Google and Amazon. The rest of the bidders are weak.
Values

Asymmetric first-price auction

– The value, \( v \), of a *strong bidder* for each domain is randomly and independently drawn from a distribution with density and cumulative

\[
f_s(v) = \frac{2v}{v^2} \quad \text{and} \quad F_s(v) = \left(\frac{v}{5000}\right)^2
\]

on the interval \([0, 5000]\), rounded to the nearest integer. The mean value then is 3750 thousand dollars.

– The value \( v \) of a *weak bidder* for each domain is randomly drawn from a distribution with density and cumulative

\[
f_w(v) = \frac{2}{v} \left(1 - \frac{v}{5000}\right) \quad \text{and} \quad F_w(v) = 1 - \left(1 - \frac{v}{5000}\right)^2
\]

on the interval \([0, 5000]\), rounded to the nearest integer. The mean value then is 1250 thousand dollars.
Bidding strategy

Asymmetric first-price auction

• We can also calculate the unique symmetric equilibrium when there are two bidders and each bidder’s value is independently drawn from a triangle distribution.

• With two strong bidders, the symmetric equilibrium bid function is

\[
b_{2\text{strong}}(v) = \frac{4 + 2v(4 + 3v(2 + v))}{15(1 + v)^2}.
\]

• With two weak bidders, the symmetric equilibrium bid function is

\[
b_{2\text{weak}}(v) = \frac{1}{10}(1 + 4v).
\]
Bidding strategy

Asymmetric first-price auction

• When the bidders’ values are drawn from different distributions then numerical methods must be used to compute the equilibrium. As an example, we present the case with one strong bidder and one weak bidder in the figure below. Notice that the weak bidder bids more aggressively than the strong bidder to compensate for the weakness; similarly the strong bidder bids less aggressively than the weak bidder in recognition of her relative strength.
One weak and one strong
Asymmetric first-price auction

![Graph showing the relationship between value and bid for weak and strong participants.](image-url)
Bidding tool
Bidding tool

• Provides generic bidding tool (Excel workbook)
  – All domains (rows)
  – Number of bidders by domain
  – Eligibility of each bidder by domains
  – Value by domain (bidder pastes her private information into tool from auction system)
  – *Equilibrium bid from one-item auction without budget constraints when known*
  – “Your bid” by domain
  – Upload integration with auction system
### Simultaneous ascending clock

<table>
<thead>
<tr>
<th>Domain</th>
<th>No of bidders</th>
<th>Value</th>
<th>Your bid</th>
<th>Equil bid</th>
<th>Minds+</th>
<th>Machines</th>
<th>Google</th>
<th>Famous Four</th>
<th>Uniregistr y</th>
<th>Afilias</th>
<th>Amazon</th>
<th>Radix</th>
<th>Fairwinds</th>
<th>Nu Dot</th>
<th>United TLD</th>
<th>Top Level Design</th>
<th>Merchant Law</th>
<th>Dish</th>
<th>TLD Asia</th>
<th>Registry</th>
</tr>
</thead>
<tbody>
<tr>
<td>.box</td>
<td>2</td>
<td>536</td>
<td>1,012</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.buy</td>
<td>5</td>
<td>2,647</td>
<td>2,431</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.coupon</td>
<td>2</td>
<td>2,110</td>
<td>1,537</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.deal</td>
<td>2</td>
<td>2,306</td>
<td>1,602</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.dev</td>
<td>2</td>
<td>3,560</td>
<td>2,020</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.drive</td>
<td>2</td>
<td>2,237</td>
<td>1,579</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.free</td>
<td>5</td>
<td>3,580</td>
<td>3,053</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.kids</td>
<td>2</td>
<td>2,391</td>
<td>1,630</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.map</td>
<td>3</td>
<td>406</td>
<td>1,036</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.mobile</td>
<td>3</td>
<td>1,549</td>
<td>1,608</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.play</td>
<td>4</td>
<td>4,908</td>
<td>3,695</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.save</td>
<td>2</td>
<td>1,391</td>
<td>1,297</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.search</td>
<td>4</td>
<td>959</td>
<td>1,325</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.talk</td>
<td>2</td>
<td>1,708</td>
<td>1,403</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.video</td>
<td>4</td>
<td>3,911</td>
<td>3,097</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.wow</td>
<td>3</td>
<td>1,295</td>
<td>1,481</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.you</td>
<td>2</td>
<td>2,541</td>
<td>1,680</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Sequential first-price sealed-bid

<table>
<thead>
<tr>
<th>Domain</th>
<th>No of bidders</th>
<th>Value</th>
<th>Your bid</th>
<th>Equil bid</th>
<th>Minds+</th>
<th>Machines</th>
<th>Google</th>
<th>Famous Four</th>
<th>Uniregistr y</th>
<th>Afilias</th>
<th>Amazon</th>
<th>Radix</th>
<th>Fairwinds</th>
<th>Nu Dot</th>
<th>United TLD</th>
<th>Top Level Design</th>
<th>Merchant Law</th>
<th>Dish</th>
<th>TLD Asia</th>
<th>Registry</th>
</tr>
</thead>
<tbody>
<tr>
<td>.box</td>
<td>2</td>
<td>536</td>
<td>179</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.buy</td>
<td>5</td>
<td>2,647</td>
<td>1,765</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.coupon</td>
<td>2</td>
<td>2,110</td>
<td>703</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.deal</td>
<td>2</td>
<td>2,306</td>
<td>769</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.dev</td>
<td>2</td>
<td>3,560</td>
<td>1,187</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.drive</td>
<td>2</td>
<td>2,237</td>
<td>746</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.free</td>
<td>5</td>
<td>3,580</td>
<td>2,387</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.kids</td>
<td>2</td>
<td>2,391</td>
<td>797</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.map</td>
<td>3</td>
<td>406</td>
<td>203</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.mobile</td>
<td>3</td>
<td>1,549</td>
<td>775</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.play</td>
<td>4</td>
<td>4,908</td>
<td>2,945</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.save</td>
<td>2</td>
<td>1,391</td>
<td>464</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.search</td>
<td>4</td>
<td>959</td>
<td>575</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.talk</td>
<td>2</td>
<td>1,708</td>
<td>569</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.video</td>
<td>4</td>
<td>3,911</td>
<td>2,347</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.wow</td>
<td>3</td>
<td>1,295</td>
<td>648</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>.you</td>
<td>2</td>
<td>2,541</td>
<td>847</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>